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THE MONIST

THE DESCRIPTIVE THEORY OF SCIENCE[1]

WE are all aware that in recent times new views have become current concerning the aim and the functions of Natural Science. We used to think that its aim was to explain. But for some time past superior persons have been assuring us that this was a complete mistake. The question which Science answers, they tell us, is not *why?* but *how?* Its aim is not at all to explain, but merely to describe.

Now if the issue here raised were, so to say, simply a domestic one, if it were just a private quarrel among the scientists themselves concerning the aims and the methods of their study, it would ill become a mere philosopher to take part in the dispute or even to offer any comments upon it. But unfortunately, although the Descriptive Theory began by concerning itself mainly with the details of scientific methodology, in these days it seems to have enlarged its scope, and has come to be a theory of the constitution of the external world itself. And whereas it began by being an obscure heresy it is now or soon will be the orthodox view: and with the whole prestige of Science to back it, will be thrust upon the layman in a thousand forms in every monthly magazine and every other evening newspaper. Thus philosophers, however ignorant of Science, have no longer the right to ignore this theory. For

[1] A paper read to the Moral Science Club, Cambridge, England.

it is entering, or has entered, upon the territory of Philosophy itself.

Yet although this theory is "very much in the air" at the present time, one has some difficulty in saying exactly what it is. Those who expound it in print seem to fall chiefly into two classes. Either they have been scientific writers like Mach and Karl Pearson, whose main interest is the study of scientific method, and whose philosophical views are stated mainly in 'asides' and intended perhaps rather to shock than to convince: or they have been popularisers (I do not use the word in an invidious sense) trying to make the latest discoveries of Science accessible to the layman, and although it is obvious enough that they have pretty definite philosophical views, those views are rather insinuated than expounded or argued for. The most important exception known to me is Signor Croce, who in his *Logica* (Part II chapters 5 and 6) seems to adopt the Descriptive Theory, though not under that name. But his account of it, though brilliantly written, is so brief and at the same time so metaphorical that I am not sure whether I have understood it at all. Nor can I quite make out how far another distinguished idealist writer, Mr. R. G. Collingwood, intends to adopt the theory. It seems to me that the conception of Science expounded by him in *Speculum Mentis* (ch. 5) is a kind of cross between the Descriptive Theory and the Pragmatist Theory. But in any case it is worth noticing that in his opinion the movement of which that view is the result is "by far the most important intellectual movement of the last half-century" (p. 181).

What is clear enough is that all these writers want us to change our opinions concerning the external world in some very radical way, but in exactly what way, remains

rather obscure. We can only gather certain hints. And in the first place, the Descriptive Theory, it seems, has abolished a number of things which most people believe in; its first triumph, we gather, was to do away with the notion of *Force*. But latterly, it seems, even *Causation* is unsafe: we hear it said that the word 'cause' has no place in Science, and consequently the old notion of a *Law of Nature* has somehow to be altered. And *Substance* of course has long ago been dropped.

Secondly, we gather that the Descriptive Theory, if it does not put Realism entirely out of court (meaning by Realism the doctrine that a world which can be called 'physical' in some not-too-Pickwickian sense exists independent of consciousness) at any rate makes Realism very much more improbable than it used to be, and makes some form of Idealism very much more probable. Thus, Mr. J. B. S. Haldane tells us in *Daedalus* that a Kantian period is upon us, and there are signs that Solipsism is becoming popular among distinguished physicists.

Thirdly, if Realism is much less probable, *a fortiori* the old-fashioned Materialism of the last century is less probable still. And it follows from this, we hear, that there is no longer any conflict between Science and Religion, or again between Science and the aesthetic consciousness. (*cf.* Mr. J. W. N. Sullivan's writings.)

Fourthly, many people seem to hold that the propositions of Science are neither true nor false, but only convenient or inconvenient—that is, that they are not propositions at all, but (I suppose) imperatives, like the recipes in a cookery book. It seems very doubtful whether this doctrine can be reconciled with the Descriptive Theory: yet the same people, I think, sometimes hold both. Obviously, it goes a long way with the Pragmatists: but as

Mr. Collingwood points out, it is not strictly speaking Pragmatism. It does not hold that 'true' *means the same as* 'convenient.' It is not committed to any particular theory as to the meaning of the term 'true,' and could quite well hold that this term is indefinable. Nor does it say that *no* propositions are either true or false (in which case it would of course be self-refuting): but only that the so-called propositions of Science (*e.g.,* those that profess to state Natural Laws) are neither true nor false, because they are not really propositions. The propositions of Philosophy, on the other hand, and according to Croce those of History also, really are propositions, and *are* therefore either true or false, according to the theory.

Lastly, the Theory of Relativity appears to have given a new impulse to the Descriptive view. Mr. Russell says somewhere that it has shaken the traditional doctrine of substance more than all the arguments of the philosophers. Some people indeed seem to think that the Descriptive view (with all the implications that I have cited) has been definitely established by the Theory of Relativity, or at least that nobody can consistently accept the one and reject the other. Other authorities deny these propositions: and it seems probable that they are right, for it would be very odd if a scientific hypothesis, however far-reaching, could establish or refute a philosophical theory concerning the external world and the nature of our consciousness of it—and that is what the Descriptive Theory really is. But however this may be, it is I believe true that many relativists are also descriptivists, and there seem to be grounds for thinking (I speak with diffidence as a layman) that the Theory of Relativity is more easily expounded in terms of the descriptivist philosophy than in terms of a more ordinary realistic philosophy, though it is compatible with

either. It is therefore psychologically natural, though not logically necessary, that the Descriptive Theory should have strengthened its hold both upon the scientific and the lay mind ever since the Relativity Theory came to be generally accepted.

These then, or something like them, are the views about Nature which the Descriptive Theory asks us to accept. In some ways they are attractive. The conflict between Science and Religion has troubled mankind for two centuries at least: and there have been lesser conflicts between Science and the moral consciousness and perhaps between Science and the aesthetic consciousness. Everybody would like to see these conflicts abolished. Mankind would then have some chance of becoming really civilised. But is not the price too great? Are we prepared to give up the categories of substance and cause, and to surrender the independent reality of the external world? Are we prepared to give up the hope of explaining the changes that we perceive and content ourselves with describing them? (If scientific investigators cannot explain them, it is pretty certain that nobody else can.) And perhaps the statement that Science is neither true nor false but only convenient is the most shocking of all.

If we are to make up our minds concerning these doctrines, we must obviously begin by asking whether they really do follow from the Descriptive Theory. Perhaps some of them do, and others do not. We must therefore enter upon a philosophical examination of the theory, and we must examine it (I repeat this in order that there may be no misunderstanding) not in so far as it is a theory of scientific method, for that is not our business; but in so far as it is a philosophy, *i.e.*, a theory of Nature and of our consciousness of Nature.

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Clearly we must begin by asking two questions: 1. What is it that is described? 2. What does 'describing' here mean?

It is fairly obvious what answer the descriptivist would give to our first question. He would say that the entities which science describes are 'phenomena' or 'sensations' or possibly 'perceptions.' These words are notoriously ambiguous. But it is pretty clear that they here mean sense-data or percepts, not sensings or perceivings. No doubt some descriptivists have accepted Idealism simply because they failed to see this ambiguity. But, as we shall see later, they have better arguments for Idealism than this one, and their theory can perfectly well be stated in terms of sense-data or percepts. Indeed I think that most descriptivists would be quite content to say that the task of science is to describe *events:* and in this word there is no reference to the consciousness of sentient beings.

The second question "what does 'describing' mean?" is much more difficult. To describe an entity is presumably to recognize it to be an instance[1a] of some universal or concept: and to express one's recognition in a proposition. The subject of the proposition will always be a singular term.[2] But I do not think it need be a *concrete* term: for instance it is a description of honesty to say that it is the best policy. It might be held that the predicate must always have the definite or indefinite article prefixed: thus *Trinity College is the largest college in Cambridge* would be called a description of Trinity College. But *Trinity College is large* would not. It is not clear to me that this view would be correct. It seems to me that to assign any

[1a] This universal may cover the whole being of the entity, as when I say this is a cat: or it may cover only an element in its being, as when I say this is black, or this is on the floor. (The distinction is Cook Wilson's.)

[2] It might itself be a description: *e.g.*, The man on the left of the fireplace is the Professor of Latin.

characteristic to an entity is so far to describe it, though of course the description need not be either sufficient or complete.

The simplest and most fundamental descriptions are those in which the subject is a demonstrative pronoun: *e.g. this is a cow, that is the Professor of Latin.* But as these two examples show, not all descriptions, even of this fundamental kind, can be called scientific. No one would say that it is the business of Science to utter such statements as *This is a cow.* And if it be said that these things (the cow and the Professor) are not sense-data, nor events, we may change our example: would anyone say that *this is a noise* or *that is a lightning-flash* were the kind of statements which Science aims at making? Clearly not. When Science makes such statements, it does so merely as a means. It does not think that its aim has been achieved when it has made no matter how many of them. Yet they are certainly descriptions, and descriptions of sense-data, or of events, or (if the word is too dear to be abandoned) of 'phenomena.'

It is more plausible to say that the descriptions which Science seeks to give are *simple* or *general: i.e.,* that it seeks to describe not one event at a time, but whole groups of events at once. And this is what descriptivist writers usually say: some of them illustrate or perhaps obscure their meaning by comparing Science with shorthand. Thus Croce says "the natural sciences and the empirical concepts which compose them arise as a shorthand transcription of the living and changeable reality" (*Logica,* Part 2, Ch. 5, p. 224). And it may well be said that *All matter gravitates* or *Salt is soluble in water* really are 'simple' descriptions in the sense that each of them describes a multitude of events at once; and that if we could find a single statement that described at one blow all the events that there

have been, are or will be, the ultimate simplification would have been accomplished, and the final goal of Science reached. (I do not think it is necessary to distinguish 'simple' from 'general.' If descriptivists usually prefer the term 'simple' or sometimes 'brief,' this is probably due to the conceptualist bias which many of them are afflicted with—a bias which we may discount for the moment, since it does not seem essential to their theory.)

Still 'simple' by itself is clearly insufficient. The statement that 'Nature is what is known by the senses' is a description of Nature, and a very simple or very general one. But nobody would say that it was a *scientific* statement. It is obviously philosophical, not scientific. Science is not content with anything so *formal* as this. The description at which it aims must somehow comprehend within itself the specific detail of Nature: whereas the statement *Nature is what is known by the senses* might still have been true even if the observed detail of Nature had been quite different from what it in fact is. It is a description of all possible 'Natures': whereas Science wants to give a description of this particular Nature which now is, and in which we live and move. If we put in the word 'comprehensive' this point is made clear. For *comprehensive* and *formal* are obviously opposed. The proper expression of the theory therefore is: what Science aims at is a description of sense-data or of events which is as simple and comprehensive as possible.

But even so we are not out of the wood. Imagine that we could take a view of all the events which have happened, are happening, and will happen. Is it not obvious that there are *many different ways* in which they could be simply-and-comprehensively described? For instance, we might take an arbitrary event say that which is happening

in the middle of this page at 8 p. m., g. m. t., on May 24th 1929: and we might describe all events past, present and future by stating their spatio-temporal relations to this. This description would be comprehensive in that it would take account of specific differences between events, and would not confine itself merely to those formal (or so to say metaphysical) characteristics which they all have in common. And it would be susceptible of 'simplification.' For space-time could be parcelled out into regions, and all the events that occurred in region No. 1 could be grouped together and so could all the events that occurred in region No. 2, and so on: and these regions could then be grouped into larger regions. But such a description, if we could make it, would not be Science in the sense in which Physics or Biology are sciences. What we should have constructed genera. But this again, whatever it is, is not Science.

Or again we might describe all these events by reference to their sensible qualities. And the sensible qualities could be grouped under a small number of heads—colour, sound, thermal quality, etc.—each having its species and sub-species: this would enable us to simplify our description. But this classification of events in terms of their sensible qualities, though it would be a simple and comprehensive describing of them, obviously would not be Science.

Again we might, I suppose, use as our *fundamentum descriptionis* the emotions aroused by these events in actual or possible observers: some of the events would be disgusting, others alarming, others merely boring, and so on. And here also our description could be simplified by grouping these characteristics as species under a small number of genera. But this again, whatever it is, is not science.

From these examples it is plain that our statement of the Descriptive Theory is not yet satisfactory. It is not enough to say that the aim of Science is to describe events

as simply and comprehensively as possible. It is necessary to specify in terms of what kind of characteristics the describing is to be done, *i.e.,* to state what we called just now the *fundamentum descriptionis.*

At first sight it seems easy to do this. Why not say 'to give as simple and comprehensive a description as possible in terms of cause and effect'? But of course this will not do. For the Descriptive Theory eschews the notion of cause and effect: notoriously, it maintains that they are mere relics of primitive anthropomorphism and have no place in Science. And it professes to provide an alternative to the ordinary cause-and-effect theory of Science: indeed to provide one is perhaps its main business.

So we must restate the *fundamentum descriptionis* and say: the Descriptive Theory holds that the aim of Science is to describe events as simply and comprehensively as possible *in terms of regular sequence.* And with this, we may hope we have at least reached a satisfactory notion of the sort of describing that is meant. If we are right, the theory will have to distinguish three main stages in any scientific inquiry.

The *first* stage is that of judgments of perception. Here we take the observed events, and we recognize that each such event has certain characteristics, or is a certain type or kind of event, *e.g.,* This is an explosion, this is the fall of an apple, that is a lightning flash, this is the taking of Prussic acid by an Englishman, and so on. (The scientific advocates of the Descriptive Theory do not usually notice this stage, I think. Or if they do, they do not see what difficulties it raises. Their theories of perception are in general very naive. Usually they are either crude sensationalists or what I may call physiological solipsists.)

The *second* stage is as follows: (a) As a result of the first stage, we have now before us a vast multitude of events, each having a certain character or having certain qualities; (b) we now notice that the number of characters is a good deal smaller than the number of events, *i.e.,* that every event *resembles* many others in many respects—there are many explosions, and many Englishmen take Prussic acid; and thus we group events into *classes;* (c) we find that each class of events has members at several or many different dates and places; (d) we now seek for each kind or class of event a determinate kind or class of constant antecedents. (This is the step expounded by Mill in his chapters on the Inductive Methods.) This process of finding is guided by no insight into or intuitive awareness of "necessary connexions." So far as we know, (according to the Descriptive Theory) there *are* no such connexions, in Nature at least. We simply examine a number of cases in which E occurs: we find that in each case A precedes it at the same determinate interval, whereas its other antecedents vary from case to case: and we find that when A is removed (all else remaining the same) E is removed also. We conclude that events of the E kind are constantly preceded by events of the A kind, at a certain determinate interval of time (whether zero or not).

The *third* stage: we have now portioned out the events with which we started into a number of types of *sequence* or *process,* in each of which an event of one kind constantly follows upon an event of some other determinate kind. In the third stage, we *classify* those types of sequence, as species, under a smaller number of generic types of sequence: *e.g.* fallings of apples to the earth, risings of the tide when the moon's position changes, and movements of the planets round the earth, are all cases of gravitational attraction. (In this particular case the man-

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ner of 'specification,' or specific differentiation of species within the genus, will be quantitative. Bodies of *such and such* a mass and at *such and such* a distance will attract each other to *such and such* a degree.) Or again the changes undergone by races of animals in captivity and by races of wild animals may both be brought under the genus 'selective process'—the one artificially selective and the other naturally. If possible, the generic types of sequence so found are to be classified again as species under a smaller number of higher genera.

And so we arrive, by these three stages, at a *simple and comprehensive classification of the constant sequences of observed events.* First come the judgments of perceptions. Then the finding of constant sequences of perceived events. Then the classifying of these sequences (as specific differentiations) under a small number of genera or general types of sequence.

Now if this is the Descriptive Theory of Science, there are certain obvious comments which we may make upon it.

1. It seems to be nothing new, but to be simply the old Uniformity Theory of Mill and others. Mill and other empiricists would say that the business of Science is to discover uniformities of sequence and concomitance in the events which we observe. It is true that they talk about 'explanation,' and explanation might seem to differ from description: we might think that to explain meant to answer the question 'why?' which the Descriptive Theory forbids us to do. But when Mill speaks of explanation he clearly does not mean this. His three types of explanation, —by resolution into parts, by intermediate links and by subsumption—are simply ways of answering the question 'how?' and the first two reduce in the end to subsumption, which appears not to differ from what the Descriptive Theory calls 'simplification.'

2. A more fundamental criticism is that the descriptivist forgets how extremely *fragmentary* our observations are.[3] Most of the sequences in whose constancy we firmly believe have never been observed from beginning to end: in most cases, perhaps in all, we observe only a bit of each sequence and 'fill in' the rest. For example, bodies near the earth fall when their supports are removed. But we often observe them falling when we have not observed the removal of their supports: for instance a tile falls on our head, though we never saw it dislodged from the roof. Again we often observe removal of supports without observing the alleged constant consequence of this: for instance if I drop a pencil out of my window on a dark night, I do not see it falling. If we stick strictly to the events which we actually perceive, adding nothing, we shall find that we have evidence for very few constant sequences indeed, perhaps for none at all. Not only the limited range of our sensitive faculties, but interruptions in their working, *e.g.,* by blinking, fits of inattention or of attention to something else, periods of non-perceptual consciousness, *e.g.,* dreaming, and of purely conceptual thinking, and perhaps periods of complete unconsciousness, *e.g.,* in dreamless sleep—all these must be taken into account. Nor may we appeal to testimony, since this would land us in a vicious circle. For if we did, we should be presupposing the constancy of a sequence, the antecedent in which has not been and presumably cannot be observed: we shall be assuming that the occurrence of certain noises or the coming into being of certain black marks on paper constantly follow upon acts of knowing, believing, and the like.

Thus if we stick strictly to the perceived we find not a simply and comprehensively describable world, but a

[3] This criticism applies equally to Mill.

chaos of undescribable fragments. This obvious fact is concealed from the descriptivists and from Mill because they tacitly suppose themselves to be describing the sense-data of a sort of *omnipercipient* observer, whose senses suffer from no limits and who can occupy all points of view at once. We all believe that if there were such an observer, the entities which he observed *would* be simply and comprehensively describable. But no one of us, not even Mach, is omnipercipient; and our notion of the experience which an omnipercipient observer would enjoy is formed by the most bare-faced extrapolation from the miserably fragmentary data which alone are perceived by each of us.

It follows that the proposition 'What we observe forms part of a simply and comprehensively describable order of Nature' is one which observation not only cannot prove, but cannot even support—just as on the ordinary non-descriptivist theory the Law of Universal Causation cannot be proved or even supported by observation. And this I think is rather a shock to the Descriptive Theory: a strict empiricist is not really entitled to hold it — yet many (though not all) of its advocates profess to be strict empiricists, and to have adopted the theory for that very reason.

We must also object to the doctrine of Croce that the business of the physical sciences is to *make an index to History* (*i.e.*, I suppose, to the perceived). This I think is just a metaphorical way of putting the Descriptive Theory. The objection to it is: before you can make this index you must forget the greater part of your text. For observation or history only gives you the most meagre fragments, and if you stick to them alone, you will have nothing to make an index to.

3. What about *prediction?* How can we describe what has not happened yet? It is very odd that advocates of the

Descriptive Theory stress the practical utility of Science so much. For its practical utility disappears entirely if you cannot describe what has not happened yet. And what an extraordinary kind of description that is! What we may call post-diction—*i.e.*, discovering what must have happened in the unobserved past, as geology for instance does, or indeed discovering what must have happened before I was born or before I came into the room—raises just the same difficulty. (We may add, for the benefit of the Croceans, that all historical research which uses documents or inscriptions raises it too.) Or in terms of Croce's own metaphor, by what right do we make an index to those parts of the book which we haven't read yet, and to those which we cannot possibly read?

It seems to me that both these objections (Nos. 2 and 3) can be met only in one way—by the use of what Kant calls a Transcendental Argument.

No. 2 might be met as follows: If what I observe cannot be thought of as part of a simply and comprehensibly describable order of Nature, my unity and identity as a self-conscious thinking being cannot be preserved. Therefore whatever *I* observe, so long as I remain myself, must be capable of being so thought of. Thus in terms of Croce's metaphor I am entitled to forge the greater part of my text, adding enormously to the few fragments which alone are given to observation. For the knowledge "that the text is indexable" is in a manner prior to the reading of the text itself. And what seems like wanton 'cookery' is really a necessary precondition of self-conscious experiencing.

No. 3 is met similarly. If what I observe now and what I shall observe in the future cannot be thought of as parts of *the same* simply describable order of Nature, I shall not be the same self-conscious thinking being in the future as I am now. Therefore whatever *I* observe, so

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long as I remain myself, must not only be *describable,* but *describable in the same manner as* what I observe now. (Analogously for post-diction.)

If these two transcendental arguments were admitted, the Descriptive Theory (thus supplemented) would I think be perfectly tenable. But we should have to say that although the business of the scientific investigator *in his scientific capacity* was just to describe events as simply and comprehensively as possible: yet in his capacity of *intelligent being or self-conscious subject* he would do far more than describe; by the process which Kant not very happily calls synthesis, he would pass beyond the meagre and chaotic data of sense, and would construct or rather constructively discover the very entities which as scientist he subsequently describes.

If on the other hand all transcendental arguments are rejected, this rejection clearly affects *all* theories of Science, not merely the Descriptive Theory. For they all have to assume the truth of some axiom or axioms which do not seem self-evident; and if those axioms cannot be proved by transcendental argument, there seems no way of proving them at all. The descriptivist is only at a disadvantage in this, that he ususally professes that his theory is empirical in a sense in which other theories are not—a claim which, as we have seen, cannot be admitted for a moment.

So much by way of exposition of the Descriptive Theory. We must now try to say something about its wider philosophical implications. And first, it is thought in some quarters that the theory has succeeded in exploding the notions of substance and of causation. I cannot see the slightest ground for this. The most it could claim to have shewn is that *Science* has no need of these notions. But scientific cognition is not the only mode of cognition: and what Science studies is not the whole of the real. And

indeed it may be said, this is just the point which the Descriptive Theory (as contrasted with previous theories) has made perfectly clear. It might be the case that in our ordinary perception of the external world (though not in our scientific describing of it) we constantly apply those categories, or apprehend those categorical factors; as indeed we obviously do. Further, if we did not, the Descriptive Theory would have nothing to describe. What it describes are *events:* and whatever account we may give of substance, and under whatever other names we may introduce it,[4] it seems to me perfectly clear that there can be no events without substances to or in which they happen. And not only cannot events be without substances; it is even more clear that they cannot be *apprehended* as events except we also apprehend the substances to or in which they happen. Moreover, in order to be conscious of the persistent identity of a substance which endures through time, we must I think apprehend relations of *immanent causality* subsisting between the events which make up its history. (Whether a substance can *have* identity in the absence of such immanent causality I do not know.) And it seems impossible to draw any line between immanent causality and *transeunt causality,* since a single substance may consist of substances, and the transeunt action of these substances upon one another may be essential to the very existence and therefore to the persistence through time of the whole which they together make up. (Any machine or any organism will do as an illustration.)

We may also bring certain arguments *ad homines* against those descriptivists who think they have removed substance and causality not only from Science, but from Reality itself. In the first place they themselves must allow that there are what we may call *descriptive correlations*

[4] For instance under the name "*long* event."

between events. And these correlations will be in at least two 'dimensions'—simultaneous and successive. A set of simultaneous and successive events united into one describable whole by such correlations looks uncommonly like what most people would call a substance—especially when we remember, as we must, that the *correlata,* the 'events' of which the describable whole is formed, are really only abstractions from a continuous process. And the descriptive correlation which binds them into one whole looks uncommonly like immanent causality. A similar argument could be brought with regard to transeunt causality.

But to this the descriptivist may answer that the only sort of substance he is abolishing is the old-fashioned *substratum*—the something we know not what which stands over against or behind qualities, relations and events, and is entirely inaccessible to observation. And he may add that the only sort of causality he is abolishing is the *active* or anthropomorphic sort. Now I shall waste no tears on the 'something we know not what' of Locke. It seems to me that the identity of a substance through the diversity of its qualities and relations, and its identical persistence through time, can be otherwise provided for—though even then, it is by no means clear to me that change (to say nothing of individuation) can be adequately expounded without the use of some conception analogous to the *matter* of the Schoolmen, Aristotle's ὕλη: and I think I can see this turning up in modern philosophy under other and to my mind less suitable names, *e.g.,* under the name of Space-Time.

But if we let go the Lockean substance, we cannot so easily surrender activity. And here we may use an *ad hominem* argument, somewhat similar to a celebrated argument of Mr. W. E. Johnson against the Naturalists.[5]

[5] *Logic,* Vol. III.

Descriptivists warn us that our ordinary notions of causality are anthropomorphic. According to them, we ordinarily believe that the earth pulls at the moon with an effort, as we tug at a recalcitrant kite. And that belief they consider to be a mistake. The earth, they assure us, does not really make efforts, and it enjoys no organic sensations. And they seem to regard it as their chief business in life to repeat these solemn platitudes *ad nauseam*. But when these same authors come as they sometimes do to dabble in Psychology, we find that they try to abolish activity in men also, just as they abolished it in the external world. They reduce the human mind with Hume to a series of sensations and images, and we discover to our astonishment that there is no 'we,' and that if there were, we could not conceivably *do* anything.

But if this is so, what becomes of the charge of anthropomorphism? If we do not ourselves experience activity, how can we possibly make the mistake of 'projecting' it into external objects? Not only would the mistake not occur, it would not even be conceivable. I cannot mistakenly identify A with B unless I have got the conception of B. And how am I to get it, except by apprehending instances of it? A similar reply may be made to the charge of *animism* which is sometimes brought against those who accept the category of substance. How can I 'project' the unity of my own ego into the external world if I am not conscious of the unity of my own ego, and how can I be that, if my ego is not a unity at all, but a mere series of sensations and images?

But to leave *ad hominem* arguments, is it not obvious that we are in some sense conscious of our own activity, *i.e.*, of *doing* things? I should have thought that there was nothing more certain than this. Thus in our own case, if in no other, the activity view of causation seems to apply.

But if this is so, I venture to suggest that we can go further. We cannot act without *acting upon* something. And in acting upon a thing, *e.g.,* in bending a stick, are we not *eo ipso* conscious if not of counter-action on the thing's part, at any rate of something very like it? I do not see how to argue the matter. The consciousness of activity in ourselves, and of this quasi-action in the things which we act upon, is either an absolutely fundamental datum, which Philosophy must just accept as an ultimate, or it is a mere delusion. I can only appeal to every man's reflection: and let him reflect (if I may say so) not when he is in the study, or even when he is playing backgammon with his friends, but when he is swimming, or climbing a mountain, or digging a hole in the garden.

This quasi-action of external objects differs from our own action in being non-teleological in many cases.[6] It is rather *obsistence* than *action.* But whatever it is, that it is something, and that it is more like activity than mere sequence (however simply describable) seems to me so certain and obvious that arguments against it must be wrong somewhere. And if things exercise this quasi-action upon us, we may fairly believe that they exercise it on one another; and if so, the uniformity theory of causation will not stand as a complete account.

There is one more question which is so important that we must say something about it. In what sense, if any, is the descriptivist committed to an *idealistic* theory of knowledge?

As we noticed earlier it seems to be thought that once we adopt this view of Science and of natural laws ('laws are only descriptions of how things happen'), such philosophical doctrines as Materialism, and indeed all forms of 'Naturalistic' doctrine, are very much weakened. The

[6] When it does display teleology, we say that the object is either itself alive, or produced by what is alive.

question is, why is this thought to be so? It is true that the strength of Naturalism, as of sin, is the law. But then the laws of nature are not in any way abolished by the Descriptive Theory: it is true that their 'necessary' character (if they were ever alleged to have it) is done away with, in a sense. It is not necessary that every A should be followed by a B, and every B preceded by an A: it just happens to be so. But if the Kantian arguments above expounded are valid, it *is* necessary that every B should be preceded by *some* one and the same determinate kind of event. It is true that the word law is avoided by the Descriptive Theory, perhaps wisely: yet its old meaning, or something very like it, is retained. And Science can still make just the same predictions on the Descriptive Theory as it could on the old theory. How then is Naturalism weakened?

Is it that the word 'description' lays more stress on the activity of the mind than the word 'law' did? A description is something that the thinker makes or creates; but a law is something that just is, whether anyone thinks of it or not. Thus we arrive at a subjectivist or idealist view of Science: matter, for instance, (we are told) is not anything real—it is just *our way of describing sensations*—and the material world is just the totality of scientifically-described sensations, actual and possible.

But why accept this view? It will only recommend itself to Nominalists, and Nominalism is false, since it cannot even state itself without giving up Nominalism—still less can it *argue* in favour of itself. But if we reject Nominalism, and hold that to describe something is not merely to utter noises, but to express in words the characters which entities are apprehended by us to possess—then the Descriptive Theory gives no support whatever to subjectivist theories of knowledge. Science will be the

description of uniformities really existing in the world, or of relations of non-separation really subsisting between events; and those uniformities will really be the specific differentiations of those generic types of uniformity under which Science classifies them. Both description and classification then are modes of apprehending what really is, or of expressing such apprehension in language. Whether the previous (and non-scientific) acts of categorial synthesis by which we 'refer' our fragmentary sense-data to an ordered system of Nature be a support to Idealism we need not discuss.[7] For *any* theory of Science, and not the Descriptive Theory only, must presuppose those syntheses.

But it will be said, what about the *Denkökonomie* of Mach (whom Croce follows)? The descriptions that Science gives are not mere *flatus vocis:* but neither, we shall be told, are they apprehensions of real relations of constant sequence and of species to genera: they are devices to save us trouble. But we must ask, what kind of trouble do they save? Is it the trouble of *thinking* that they save (as Mach expressly says in one place, and as the term *Denkökonomie* strongly suggests), *i.e.,* I suppose the trouble of thinking of each event after another, one by one? But wouldn't it be less trouble not to think at all? And if you are going to think, the only labour-saving device that is any good is one that will help you to apprehend that which really is; the only genuine thought-economy is the apprehension of *real* universals and *real* relations. To pretend they are there when they are not will not help you to know the real, and is therefore no economy at all. Or is it that the descriptions given by Science save us trouble in *acting?* Certainly they do. But they would not save any trouble at all unless they were *cum fundamento in re.* Mere fictions and fairy tales do not assist us to act more

[7] For my own part, I do not think that it is.

successfully. But apprehension of the real does, because it is the real that we have to act in and upon. And if it be said "surely Science uses fictions (*e.g.*, the idea of a perfect gas) and 'mechanical models' (*e.g.*, of the atom as a sort of solar system) and mathematical devices of which no rational account can be given"—if this is said, we answer by asking 1. Why does Science do this? 2. Is it *describing* when it does this, and if not, what is the good of mentioning these questionable procedures? Of course the truth is that these are not *part* of the description itself, but *means* to it. And the veracity of the description is not affected by this, any more than it is by the fact that the scientific man's microscope was paid for by a fictitious cheque.

There seems to be only one way left in which the descriptive theory of science could be held to have weakened naturalistic doctrines. We have referred to the point before in another connection. If we hold that the business of Science is just to describe the course of Nature, we shall be led to reflect that the same entity can often be described in many different ways, all equally correct and legitimate. What can be described in one way upon one *fundamentum descriptionis, e.g.*, by the classification of regular sequences, can equally properly be described upon another, *e.g.*, in an aesthetic or historical or geographical way. Now perhaps naturalistic philosophers have thought and still think that Science is *the* one and only way of apprehending Nature. And certainly the habit of speaking of scientific propositions as *the* laws of Nature may have encouraged them in this opinion. Or to put the same point rather differently, perhaps they have not always realised that the proposition "Science is the discovery of the laws of Nature" is in one sense just a tautology, since *Nature* in one sense just means that whose laws Science discovers. If we adopt this language we must say that the naturalists

believe that Nature is the whole of the Real, and that this is a mere prejudice since (1) that which is Nature may also be something else (*e.g.*, it may be a beautiful spectacle, or an object of religious awe); (2) there may be and there are other realities besides this one which is Nature (*e.g.*, minds). And it might be held that the Descriptive Theory of Science helped to dispel this prejudice.

If this were all, the Descriptive Theory would perform a certain service to anti-naturalistic philosophers, but not a very important or a very new one. But it seems plain that those who advocate the theory consider that its services in these other respects are very important indeed. And it is very desirable that they should (if the phrase may now be allowed) explain why they think so.

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MATHEMATICS AND EMERGENT EVOLUTION

I. *Causal Laws and Differential Equations*

EVER since the time of David Hume the problem of causality has been a source of worry to philosophers. Attempts to reëstablish certainty in the inductive sciences through a refutation of Hume's reasoning have not met with general acceptance. We may therefore conclude that the question of necessary connection of natural processes is still an open one.

One of the most promising of the suggested solutions lies in the attempt to assimilate the notion of causality to the mathematical concept of functional relationship. According to some theorists, the cause-effect relation may best be regarded as but another name for the mathematical relation of dependent variability. Thus, to illustrate, if we have a ladder leaning against a wall, the cause of the descent of the top of the ladder to the base of the wall, which common sense ascribes to the fact that someone pulls the bottom of the ladder away from the base, may be interpreted mathematically as a case in which a variation of the independent variable, x, (distance from the origin along the abscissa of the Cartesian coördinates) is correlated with a corresponding variation of the other (dependent) variable, y, (or distance along the ordinate). Here there is a causal connection because $y=f(x)$.

This mathematical interpretation of causality has not

received the attention it deserves. A consideration of its utility in modern science may therefore help emphasize its value. Moreover, giving this viewpoint a more specific setting in the context of modern science has the additional value that it serves to illustrate the way in which science is so inextricably bound up with mathematics that progress in one calls for and creates progress in the other. As an illustration of the evolution of this notion of cause as a phase of the development of mathematical technique we will examine the way in which modern physical science has utilized the notions of the infinitesimal calculus.

Physics aims at predictability. Prediction in physics occurs when we judge the future of a system from our knowledge of its past. But in order that a system can get from where it is to some other (future) state, it must pass through the intervening points (distance) in space and time. In order to be able to predict, physics finds it desirable to employ a mathematical technique which will enable it to state the direction of change at each moment of the process. This technique was discovered by Newton and Leibniz at about the same time, and, as research seems to show, independently. In this type of mathematics, where differential and integral equations are used, such fundamental problems of dynamics as that of finding the velocity of a particle at a given instant are handled. Since the problem of finding the "tangent" at the point of a curve and of finding the velocity of a particle describing this curve when it gets to that point are the same kind of problem, the methods of calculus will yield an answer.

The doctrine that the processes of nature are predictable is called determinism. The deterministic theory states that the future history of the universe (either as a whole or in part) can be read from a *complete* description of its present state, including here a knowledge of the laws of

change which are appropriate to the system. Such predictability means, of course, that nothing new or unforeseen shall enter to upset the calculations. In a deterministic world all causes are *additive;* all effects are *resultant* effects. But in a world of emergent evolution such *a priori* predictability is impossible.

Now it is precisely in infinitesimal calculus that we reduce to a minimum the temporal interval between cause and effect, thus decreasing the possibility of qualitative novelties. By putting the laws of sequence or behavior in the form of differential equations, in the above fashion, the scientist provides himself with a guarantee that the axiom "there is no more in the effect than there was in the cause" will be observed. This view finds support in the statement of P. W. Bridgman[1] when he tells us that those properties of a system which can be described in terms of linear differential equations have the property of additivity: the effects of a number of elements is the sum of the effects separately, and no new properties appear in the aggregate which were not present in the individual elements.

This reduction of the principle of causality to the mathematical concept of functional dependence has another implication in addition to that of expressing determinism and predictability. It further undermines the animistic notion of causality as the expression of purposive agencies.

[1] *The Logic of Modern Physics,* p. 221.

In his book Prof. Bridgman has previously pointed out (pp. 111-113) that in his opinion the existence of conservative functions is involved in the possibility of describing natural phenomena with differential equations. In the integral of a differential equation certain constants appear which are determined by the initial conditions, and are therefore the same during the future motion of the system. Obviously (as he says), these constants correspond to conservative properties. And in other systems, in which we may not have differential equations, we may look for something analogous to potential energy which shall measure the displacement of the system from its initial configuration. Here again conservation may be guaranteed, so that the law of the conservation of energy is preserved; for, as Poincaré has pointed out, if we have a system in which conservation apparently fails, we merely invent a new form of potential energy to save the principle of conservation.

In the teleological notion of causality the cause must precede the effect because the end or purpose precedes the process whereby that purpose or goal is realized. But in the newer view, to the extent that the time element in science can be made reversible in the equations of physics to that extent the common sense notion that the cause must precede the effect is abandoned. This is exactly what happens in relativity physics, which appears to carry still further what we might call the "deanthropomorphization" of the causality concept. In relativity physics we have the three coördinates of Cartesian geometry tied up with the time dimension in one equation. Here time is turned into space merely by giving it a minus sign. From this point of view "past" and "future" may be treated as mathematically equivalent, and causality becomes a symmetrical relation. It is this which presumably justifies the statement of Prof. G. N. Lewis,[2] according to whom relativity geometry "removes the last foothold in the physical sciences of the concept of temporal causality."

Not all investigators, however, are prepared to agree to this. H. Weyl,[3] for example, holds to the notion of causality as bound up with the unidirectional character of time from Past → Future. On the other hand, A. S. Eddington[4] appears to agree with the interpretation. He tells us that while cause and effect are clearly bound up with time's arrow, so that the cause must precede the effect, the relativity of time has obliterated this order. As he points out (p. 102), in physics the temporal relation is arrowless and there is, therefore (p. 66), no more distinction between past and future than there is between right and left. The only exception to this rule is the second law of thermodynamics.

[2] *The Anatomy of Science*, p. 134.
[3] *Space-Time-Matter*, Eng. trans., p. 310.
[4] *The Nature of the Physical World*, p. 295.

II. *Emergence, Probability and Chance*

There are a number of philosophers and scientists who have objected to the view that nature is predictable on the basis of a knowledge of physical laws; these thinkers do not believe in the mechanistic doctrine that the entire course of nature is predetermined by physical laws. The mechanistic view has been opposed on various grounds. In philosophy Bergson has voiced his objections in the thesis that nature is creative, so that new things appear, especially in the realm of biological evolution. In science a recent reason for objection to the mechanistic view has been found in the new physics, especially in the new quantum theory. Since in this field fact and opinion are still fluid, any appearance of finality or dogmatism of statement is to be avoided. But it is at least clear that our ideas must be reorganized in the light of newly discovered facts, so that an inquiry into the nature and meaning of such terms as cause, time, emergence, chance, etc., is very timely. The following suggestions represent but a first approach to such a consideration.

Considering first of all Bergson's attitude, we find that he objects to a mechanistic (deterministic) view for the following reason. According to Bergson, a mechanical universe is one in which time does not count. In a mechanical world time plays the rôle of an independent variable. A strictly mechanical universe would be one in which the processes could be reversed. All that can happen in a mechanistic universe is the redistribution of mass particles in space. In other words, time is spatialized. This follows inevitably, it is said, from the treatment which time receives in the equations of motion of a system. But, Bergson says, in such a view, where time does not count, the creative aspect of evolution is being ignored. Real time must not be regarded as a "phantom" of space;

true time, or duration, is the movement wherein the past "gnaws" into the future.

Considering this conception, we may note that it is true that the spatial coördinates of a system are ordinarily treated as the dependent variables, while time is considered as the independent variable. This is the case, for example, in differential equations where the second derivative is acceleration. But in general the situation is not quite so simple. There are many equations in physics in which time functions as the dependent variable. Hence even physical time has no single and unambiguous meaning, and therefore to say that time must necessarily function as an independent variable is inaccurate. In so far, then, as Bergson's views rest on this misconception, his sharp contrast between the world of matter (the inorganic) and the vital world of creative evolution is to be looked upon with suspicion.

As already intimated, those who are sympathetic with the view that the processes of nature are creative have in recent years found added reason for doubting the validity of the mechanistic interpretation. According to some investigators, one possible implication of the new physics is that it lends support to the concept of emergent evolution. Let us see how this conclusion is arrived at.

In recent years, with the advent of new theories concerned with phenomena in the realm of micro-physics, it has become increasingly difficult to apply the laws of classical dynamics (of a deterministic system). The reason for this is as follows. According to the classical theory of mechanics, if the initial conditions of a particle (or system of particles) are determined one can, by applying the appropriate laws, predict what the later position of the particle (or system) will be. But research in the domain of intra-atomic phenomena indicates that if you

specify the velocity of an electron you cannot specify its position, and if you state the position of the electron you cannot state its velocity. But for prediction both are necessary. Predictability is made more difficult by the fact that if you observe a phenomenon (e.g. a moving electron) that phenomenon is no longer what it was, for the light ray which was implicated in this reaction between the observer and the observed has been responsible for a change in the phenomenon observed. Thus the conditions demanded by classical mechanics for prediction cannot be met. Hence, it is concluded, rigid causality and determinism are impossible in physical science. The most that can be done is to predict the limits within which the electron will probably be found. At best we get statistical certainty. Some investigators have been inclined to regard this intrusion of unpredictability and chance into physics as indicative of a genuine element of indeterminism in nature. Eddington, for instance, suggests that this fact that you get merely statistical certainty and determinism in some physical processes is a step in the direction of freeing human action from the sweep of mechanistic interpretation.

This is of course a possible point of view. But there is another interpretation of the situation. According to some thinkers the use of statistics, due to the inability to study and predict individual phenomena, is not proof of indeterminism. It is witness only to our ignorance of the causal laws involved. According to this view, everything in nature is equally "necessary" (i.e. causally determined or functionally related to antecedent and concomitant events), so that "probability" and "chance" are only names for our degrees of ignorance. In this view it is supposed that we are unable to make certain predictions because we are ignorant of the causal laws involved or because the

factors operating are too numerous or too intricate for isolation and quantitative statement. In terms of the mathematical notion of causation "chance" and "novelty" arise because of our inability to specify enough coördinates to determine the course of a process.[5]

It is possible that we will always be ignorant to some extent, in the sense that our knowledge will always be fragmentary and incomplete. To say that if we knew all the facts or all the causes we could predict the properties and behavior of anything in nature is a redundancy, for here "knowing all" is synonymous with the ability to predict completely, and in cases where such prediction is impossible we can always say that this is so because we don't know everything. It is even doubtful whether in such cases the word "all" has a valid meaning, for to know all one would have to be familiar with the entire past history of the universe (if it has a "history") and this is obviously impossible.

In terms of mathematics this means—if Prof. Bridgman is right—that whenever non-additive (or emergent) effects appear we must abandon the view that nature can be completely described in terms of differential equations of the type indicated. In transitions from one "level" to another a different type of mathematics must be employed, probably that of finite difference equations. However, when we have reached the "new" level, where uniformity again reigns, it is again possible to employ calculus. Thus differential equations are employed in expressing the laws of psychophysics, but this does not mean necessarily that because the psychologist uses the same mathematics which the physicist does therefore psychological laws are reducible to the laws of physical reality. This interpretation

[5] In connection with this problem the reader should consult W. F. G. Swann's paper on "Physics and Vital Processes," in *Science*, 1928, Vol. LXVIII, pp. 413-419, and P. W. Bridgman's exposition of the "New Vision of Science," in *Harpers* for March, 1929.

appears to be in harmony with that of C. D. Broad, as summed up in the following statement: "On the emergent theory we have to reconcile ourselves to much less unity in the external world and much less intimate connection between the various sciences. At best the external world and the various sciences that deal with it will form a hierarchy. We might if we liked keep the view that there is one kind of stuff. But we should have to recognize aggregates of various orders. And there would be two fundamentally different types of law, which might be called 'intra-ordinal' and 'trans-ordinal' respectively. A trans-ordinal law would be one which connects the properties of aggregates of adjacent orders. A and B would be adjacent and in ascending order, if every aggregate of order B is composed of aggregates of order A, and if it has certain properties which no aggregate of order A possesses and which cannot be deduced from the A properties and the B-complex by any law of composition which has manifested itself in lower levels. An intra-ordinal law would be one which connects the properties of aggregates of the same order." Broad's distinction between the two types of laws would reappear in our doctrine in a distinction between the two types of mathematics employed, a mathematics of continuous quantities and a mathematics of discontinuous quantities.

If we thus interpret "emergence" and "chance" as names for our ignorance the doctrine of emergent evolution ought to prove acceptable to everyone. But such an interpretation hardly justifies the theistic interpretation which Lloyd Morgan, or the vitalistic view which Bergson, has arrived at as a result of surveying the facts of the "creative advance of nature."

[6] *The Mind and its Place in Nature*, p. 77.

III. *Matter, Energy-Fields and Emergence*

And now we come to the positive philosophical implications of the foregoing doctrine. So far as these are concerned, the writer sees no reason to modify the view previously set forth,[7] that there is a duality of natural law which allows us to set down certain contrasts between what are termed *dynamical laws* and *statistical laws*. It is interesting to note that Eddington favors such a dualism, which he expresses by the distinction between what he calls primary laws and secondary laws.[7a] The contrasts he enumerates parallel in several respects the contrasts which the present writer has noted. According to Eddington, a primary law holds for the behavior of individuals and is indifferent to time direction; whereas a secondary law holds for aggregates rather than individuals, expresses probable rather than necessary results, and therefore introduces "chance" into our picture of nature. The second law of thermodynamics, illustrating a secondary law, gives to time the arrow which indicates direction and irreversibility.

In the present view statistical laws are interpreted in a phenomenological sense. By "phenomenology" is here meant a study of that which displays or exhibits itself; it is the "descriptive" point of view in science. A phenomenological (or phenomenalistic) presentation is a pattern given in perception. Those who have followed the development of *Gestalt* theory have seen this view intrude itself into psychology. Another illustration of this attitude presents itself in the second law of thermodynamics. In botany we also find an example. Like the biochemist, who studies the intimate and complex mechanisms, we may analyze the petals of flowers into their intricate molecular

[7] *Cf.* my article, "Probability, Natural Law, and Emergence," in the *Journal of Philosophy*, 1926, Vol. 23, pp. 421-435, and "A Spiritual Behaviorism," in *The Monist*, 1927, Vol. 37, pp. 289-308.

[7a] *Cf. of cit.*, p. 75.

structures. But we may also view the flower as a phenomenal pattern. We must never forget that the flower, in spite of its detailed complexity, possesses a phenomenal simplicity which is responsible for its beauty to us.

According to the present view, it is the second law of thermodynamics which underlies the production of what are termed macroscopic (or phenomenological) patterns. This interpretation is not explicitly stated in Eddington's exposition. And yet what we have here set forth is quite consistent with his view. Eddington tells us (pp. xiv-xv) that the external world has become a shadow world, and that the drama of familiar life, from the point of view of present day physics, is a shadowgraph performance. And now we ask, why should we not identify this "shadow world" with the phenomenological world which expresses itself? And why should not the second law of thermodynamics be taken as providing the physical basis for these macroscopic patterns? Eddington almost comes upon this view when he later tells us (p. 105) that entropy is to be placed alongside beauty and melody, because it meets the essential requirement that all three appear as features of *arrangement.* Thus nature produces new (or second order) simplicities out of complexities of parts.

The writer has been arguing for years that the fields of force which are associated with material aggregates are implicated in the *averaging* process whereby macroscopic states are produced. This view, which allows us to state that *energy is the soul of matter,* appears to be in harmony with the latest surmises, among the more interesting of which we have the views of Sir J. J. Thomson as stated in his book, *Beyond the Electron.*

Basing his views in part on the results of his own son, G. P. Thomson, Prof. Thomson points out that a moving electron is a much more complicated thing than a small

point charge of electricity in motion. A moving electron is always accompanied by a series of waves, and these waves have complete control over its path; the electron is compelled to follow the lead of the waves. In other words, *the electron provides its own ether in the form of waves.* If we generalize this notion we arrive at the view that all material aggregates carry with them some sort of energy-field, which for psychology may have the significance that it provides a physical basis for what W. Köhler has pointed out: that parts of the brain which are structurally disparate may be functionally unified into a whole or *Gestalt.*[8] This view that energy-fields in the ether, or wave patterns which *are* the ether, constitute the non-material substratum which is the "soul" of "matter" is consistent with the doctrine of the author[9] that radiation plays some fundamental rôle in the fabrication of consciousness in the body.

IV. *The Bankruptcy of Materialism*

Various thinkers have noted the way in which problems may be created by the way in which we formulate our definitions. In connection with the mind-body problem this is illustrated by the fact that in classical physics matter was defined as the seat of *primary* qualities; hence, by definition, body could not be identified (or even correlated) with a conscious experience, such as color, which was therefore defined as a *secondary* quality in the mind. Thus the dualism of mind and matter has been created through *definition by postulation* and is not a result of *definition by inspection* (or induction), which is the proper type of

[8] By developing the notion of the relativity of fields of force the writer has tried to unify physical and psychological relativity (of *Gestalt* theory). This attempted synthesis appears in the *Psychological Review* for May, 1930.

[9] "Light, Wave-Mechanics, and Consciousness," *Journal of Philosophy*, 1928, Vol. 25, pp. 309-318.

It is interesting to note that a similar view has been advocated by L. L. Whyte in his booklet, *Archimedes or the Future of Physics.*

definition for a natural science to employ. Our thesis, therefore, is that the new physics now provides us with a theory of matter in which the gap between consciousness and matter is lessened. Hence the feasibility of identifying consciousness with its bodily conditions.

According to the present view, the dualism of consciousness and body is simply the (relative) dualism of matter and energy-fields. If therefore we soften the line of demarcation between matter and empty space we bridge the gap between soul and body. In order to see how traditional physics did indeed exaggerate the distinction between matter and the field of energy we need go back no further than the distinction between matter and space as embodied in Maxwell's equations. These famous equations, written in terms of calculus and therefore emphasizing continuity, were devised to express the way in which electromagnetic waves are propagated through the ether, or space free from matter. According to this view, matter might appropriately be defined as the region in which Maxwell's equations do not hold. A particle might even be defined as a hole in the ether. In either case the universally postulated sharp distinction between substance (matter) and free space (the arena of action and the seat of energy) was provided for.

These fallacies of "misplaced concreteness" and "simple location" were so firmly established in modern physics that they were presupposed in the earlier types of the modern electron theory which replaced the older atomic theory. The attempts to ascribe a definite shape and diameter (but not color, etc.) to the electron are illustrations of the continued influence of the "mental set" appropriate to naive materialism.

In philosophy, as is well known, the first definite attack

upon this theory begins with Bishop Berkeley. Contemporary philosophers completely free themselves from slavery to conventional habits of thought when—carrying Hegel's idea that *a thing is the law of its states* still further—they recognize that the concept of anything is simply the law of its behavior: *a thing is what it does.* It must be noted, however, that the properties which a thing exhibits depend as much upon the environment the thing is in as upon the thing itself. The "thing itself" is simply the more or less permanent possibility of behaving in specific ways in different environmental settings.

The scientific revolt against the materialistic view came later on, and the emancipation has come from a different route, experimental rather than epistemological. The discovery that the mass of an electron is not the same always and everywhere, but is a function of velocity, was a serious blow to the materialistic theory of substance. For if your definition of matter is put in terms of mass, and it is subsequently discovered that as the velocity of the electron increases the mass also increases, then, as R. A. Millikan has pointed out, *matter is actually being created!* Thus the old law of the conservation of matter goes overboard.

A second blow to the materialistic theory was dealt when Einstein showed that energy and matter are not two mutually exclusive entities, but are related to each other in an intimate manner. This association has become so contagious that in order to express the mutual interdependence of the two realities such terms as the "dematerialization of matter" and the "materialization of energy" are resorted to. The relation between matter and energy has indeed become so reciprocal that it reminds one of those South Sea Islanders who are said to make their living by taking in each other's washing!

The last, and probably fatal, blow was struck when experiments showed that each of these two supposedly independent forms of physical reality, matter and energy (radiation), possess properties hitherto reserved exclusively for the other. The overthrow of materialism is completed when the view is adopted that what we formerly considered to be a particle has some of the properties of waves, while what was formerly regarded as a purely wave phenomenon (radiation) behaves in some respects like a corpuscle. Such a view is disconcerting, for, as C. J. Davisson[10] remarks, this is like saying that cats are rabbits and rabbits are cats.

This evolution of the concept of the elementary particle may be summed up in the statement made years ago by Karl Pearson: *matter is non-matter in motion.* It is now evident that whatever our new theory is to be (whether Einstein's unitary field-theory or some other), the notion that matter is something sharply delimited from surrounding space must be discarded. When an important fact needs to be kept in mind there is everything in having a name for it, as Mrs. Ladd-Franklin has pointed out in another connection. Hence the timeliness of the suggestion of A. S. Eddington that we coin the term *wavicle* to express this mutual infectedness of "waves" and "particles."

As is now generally known, the first suggestions towards such a synthesis came from L. de Broglio, who advanced the idea that every mechanical phenomenon is in some sense a wave phenomenon. This analogy between mechanics and optics was developed and made more exact by Ernst Schroedinger in the doctrine of wave mechanics. Various physicists have pointed out that there are some aspects to this doctrine of wave mechanics which are still

[10] "Are Electrons Waves?", *Journal of the Franklin Institute*, May, 1928.

unsatisfactory, and that the view will have to be modified in the future. But the fundamental idea, that matter and wave phenomena must somehow be assimilated to each other, is not likely to be abandoned, though the particular way in which this underlying unity of physical phenomena is to be stated may be altered in some respects. And it is this permanent significance of wave mechanics which justifies the philosopher in trying to sift out the broader implications of the view.

The way in which energy-fields are concerned in the manifestation of consciousness will be dealt with elsewhere. We will now only point out that mental phenomena are conceived to stand in relation to physical processes as a water wave does to the molecules which compose it. Here new phenomena appear which make possible the application of the differential equations as employed by hydrodynamics, even though the "material" entities of which the liquid are constituted are discrete. The averaging effect which produces the macroscopic behavior involves an interlacing of the atomic and molecular fields of force. It must be kept in mind that for us the term "ether" is but a name for the fact of dynamic interaction of parts, and that the second law of thermodynamics (which can be interpreted as an expression of the tendency of the ether to gain energy at the expense of matter) is here understood to be a name for the tendency of material particles to come to some common basis of functional pattern.

As here interpreted, all emergence takes place within the domain of physical reality, or the space-time-matter manifold. It should be pointed out that in taking this stand we are not attempting to represent in three dimensional space the properties of psychic reality. This means, for example, that the experience of a color quality is held

to be identical with the total bodily condition of that quality. But while a secondary quality is thus conceived to exist only in the head (of the perceiver) and not in the external world, and in that sense is "subjective," it is nevertheless held to be in and of the physical world—specifically that part of the physical world which is the brain.

Human consciousness may be regarded as a new dimension, and in that sense as an emergent from human bodily processes. As has been noted by others, the wave which we see moving over the surface of the water is composed of molecules which move in but one direction—up and down—and yet the crest of the wave moves forward in a new dimension—horizontally. May not consciousness itself be regarded as such a phenomenon, in the sense that it emerges out of the synthesis of neuronic frequencies and represents the constellation of ether-patterns accompanying cortical rhythms? The "ether" need not be the ether of the electron. In an electromagnetic field of force the electric and magnetic vectors are at right angles to each other. Perhaps the ethereal components of brain processes are perpendicular to the coördinates of the manifold which represents the cortical processes as objectively viewed. If wave mechanics requires six dimensions to explain the radiation of light by an atom, is it likely that the psychologist can get along with fewer dimensions? I think not.

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MATHEMATICAL ANTINOMIES

FOR as long as we know, men have used rational processes in order to arrive at truth or to support their convictions, and have thus relied upon the validity of reason. And for as long as we know, though more infrequently, other men have attacked these arguments by showing that their opponents, or rationalists in general, must arrive, by the use of reason, at two conflicting conclusions. Such a result is a paradox, with which we have to deal in this paper.

Paradoxes are a much more serious problem for the philosopher than is generally supposed. It is not enough to consider a paradox as a child's game, or as a rather interesting form of puzzle; and to ignore it as a thing apart. For if there is only *one* paradox which cannot be broken down, reason can no longer be trusted and we can only revert to a complete scepticism. Rationalist philosophers of the past have realized this, and have attempted, with varying degrees of success, to explain the paradoxes of Zeno, which for almost twenty-four centuries remained the only paradoxes worthy of serious consideration. But within this last century there has been developed a series of mathematical antinomies which so far have been ignored by practically all philosophers. These, therefore, constitute our main problem.

Paradoxes, on inspection, group themselves into two classes: those in which there is a definite logical contra-

diction, and those which simply violate our more fundamental conceptions. The former alone are true antinomies.

Antinomies may be broken down directly if one or both theses are falsely urged, or may be converted into a paradox of the second type by the challenging of one of its premises. As the premises of a worth-while antinomy are important and unsuspectingly accepted parts of human belief, it is necessary not merely to challenge, but to propose a satisfactory substitute for the challenged premise.

A paradox of the second type may be disproved directly; or it may be shown that there is only an apparent contradiction in its conclusions; or finally, we may accept the paradox and revise our outraged preconceptions.

The most important of the antinomies which are unsolved at the present time are those dealing with various aspects of the mathematical idea of infinity. They are as follows:

The series of all ordinal numbers, which must obviously be a well-ordered set, has for its type an ordinal number greater than any in the set, yet all ordinal numbers are in the set. Similar to this is the paradox of cardinal infinity; that the set-of-all-sets must include all its subsets as elements (as they are also sets), and yet the set-of-all-subsets-of-the-set-of-all-sets, which can be put into one-one relation with part of the set-of-all-sets, is nevertheless greater than the set-of-all-sets (by Cantor's proof). And finally, the set-of-all-sets-which-do-not-include-themselves, includes itself if it does not include itself, and does not include itself if it does include itself.

The simplest and easiest way to attack these antinomies is by denying that infinity exists. It is not only simple, it is too simple. For it is not enough to say that the subject of a valid paradox does not exist. Reason must apply not only to existence, it must apply to its own, that

is to rational systems. Indeed, the latter is much more fundamental than the former. Nature may possibly be irrational, but that rational systems themselves should be irrational is utterly inconceivable. To deny the existence of infinity does not remove, but only augments our difficulty. And secondly, if we are to deny infinity (method two above), we must show that everything is as satisfactorily explained without as with infinity. It is this that philosophers have frequently ignored; and Bertrand Russell has thus been led to protest that most philosophers who have considered the problems of infinity do not even know what infinity means.

Bearing in mind the necessity of altering our premises rather than denying them, a number of possibilities occur. For example, we might redefine set in such a way that all sets include their subsets. Then all sets include themselves and the last antinomy cannot arise.[1] But the most interesting solution of these antinomies is the "Doctrine of Types" proposed by Bertrand Russell, which we will now

[1] A good reason for so doing can be found in the ordinary usage of the concept "set." A library is a set of books. The owner of a library is asked whether it contains the *Encyclopedia Britannica;* he answers in the affirmative. Yet, strictly speaking, he should answer "No, but my library contains all the books which the *Encyclopedia* contains," for the *Encyclopedia* is not a book, but a set of books, and a library is not a set of sets of books. Similarly the owner of the library will say that it contains Plato's *Republic* or Parmenides' *Fragments,* yet really the library contains books which in turn contain those works.

And we may not avoid this conclusion by considering a library as a set of printed works, rather than of individual volumes; because even then there will be many contradictions between the way we use the terms in philosophy and the way we do in life. The *Fragments* of Parmenides do not constitute a work, yet does not the library contain them? And if the *Encyclopedia* is taken to be a work, does not the library contain the volume FAL-FYZ?

The set of books which we call a library, contains, then, not only the books within it, but the sets of books, the works, the bindings, the pages and the paper of which they are made, and the words on the pages. And it follows also that there is no definite number of elements in a set. My library may contain a thousand books, but five hundred thousand pages, a hundred million words, but only four types (folio, quarto, octavo, duodecimo). If the whim seizes me to count in that way, my library contains five elements, the subset of brown books, of black books, and of red, green, and blue. And if I refuse to analyse it in any way, my library consists of only one element, the library itself. That is to say, the set contains itself.

consider. Our approach to the theory is entirely different from his, but the results are identical.

It is both obvious and accepted that we cannot define anything in terms of itself. To do so is to invite one of two possibilities: either that we have not limited ourselves in any way, or that we have limited ourselves too much. If I say that houses are houses, I have limited myself not at all, and everything can or can not be a house; if I say that houses are everything except houses, I have limited myself too much, and if anything is a house it is not a house, and if it is not a house it is a house. Mathematically, if I define x as x, $(x = x)$, then I am unlimited, and anything can or can not be x. If I define x (x being limited, say, to ± 2), as $-x$ $(x = -x)$, then if x is $+2$ it is -2, and if it is -2 it is $+2$. Logically, if I say: "This statement is true," then it either can or can not be true. If I say "This statement is false," then if it is true it is false, and if it is false it is true. The antinomy of infinity is exactly parallel to this. In the case of the set-of-all-sets-which-include-themselves, it either can or can not include itself. In the case of the set-of-all-sets-which-do-not-include themselves, if it includes itself it does not include itself, and if it does not include itself it includes itself. Our rule is that *a thing cannot be defined in terms of itself*, or that *the domain of a thing cannot be a function of the thing*, or that *a thing must be defined before its functions are defined*. These three forms of the rule are perfectly equivalent.

Now, when we speak of the set-of-all-sets, we violate this rule. For the domain of the set-of-all-sets is "sets," and the latter is a function (universal-particular) of the former. Or, the set-of-all-sets being a function of "sets," the latter must be defined before the former. But we can-

not define the latter, as a universal, until the former is defined.[2]

These considerations lead us to a reconsideration of the concept "set", in order to prevent the occurrence of this violation of our rule. A set is a function of its elements, and therefore "elements" must be defined prior to "sets." It follows that a set cannot be an element, the latter having been defined prior to the former. Having defined this element$_0$ and set$_0$, however, we may proceed to define a new term, element$_1$, as including both elements$_0$ and sets$_0$. Then we may define set$_1$ as a set of elements$_1$, and so on forever. Thus set$_n$ can include elements$_0$ and sets of any type from set$_0$ up to set$_{n-1}$, but may not include sets$_n$. No set, therefore, includes itself.

The method we suggested before as an example was to define sets so that they always include themselves. Russell's is the exact opposite. Both methods are open to this objection, that they only avoid, not settle, the issue. If Russell and his followers speak only of the set-of-all-sets-of-the-nth-degree, and never of the set-of-all-sets-of-all-degrees, they do not need to consider the antinomies. But it does not seem to be impossible to consider sets of all degrees. Such a thing seems to have a perfectly definite meaning, and if so, we are at liberty to consider the set-of-all-sets-of-all-degrees, and all the old antinomies rearise. And as long as this possibility remains, the rationalist problem is not solved. It is necessary not only to show that the antinomy can be avoided, but that it cannot occur.

And if our ordinary views as to the nature of classes are correct, it would seem that it must arise. A set, or

[2] Lest it be thought that I have contradicted myself, I should explain that either "sets" or "set-of-all-sets" may be considered the function of the other. In the first place, "sets" is a universal, and therefore a function of its particulars, of which the set-of-all-sets is one. Secondly, a set is a function of its elements, and the elements of the "set-of-all-sets" are "sets."

class, consists of those things corresponding to a particular definition; i. e., the set of x consists of those things corresponding to the definition of x. We have thus defined "set." Corresponding to this definition there is a set, the set-of-sets.[3] Thus is reinstated the antinomy of cardinal infinity. Now this set includes itself. The set-of-any-natural-object does not include itself. Thus there are sets which do and do not include themselves, and our third antinomy likewise reoccurs.

From this it is apparent that, given the usual connection between "class" and "definition," we must still encounter our paradoxological difficulties. And simply to change this definition or connection is not enough; we must show that it is faulty.

This then is what I am attempting to do. I claim that the ordinary conception of "class" or "set" is illegitimate, because it is a confusion of two distinct things, two distinct types of set.

The first type is that generated by the enumeration of its elements. This type I will call "closed set" and designate by the symbol O. The second type is that generated by a definition and which cannot be generated by enumeration,[4] for which type I will use the name "open set" and the symbol C. In the case of a O it can be definitely stated that these (enumerated) things are included in the

[3] Definition of natural object (x).....Set of (X)
Definition of "set".....Set of "sets"

[4] Which has not been generated by enumeration. The set of all printed books is to me an open set; irrespective of the theoretic possibility of listing all printed books. When someone publishes a complete index of all printed books, and shows that none have been omitted, then the set of all books will be a closed set. That it makes a real difference is shown by the following argument. At present, I can definitely say Ueberweg's *Geschichte der Philosophie* belongs to the set of all books—the book lies before me on the table. I can say definitely that Socrates' "Meditations sur la Philosophie de Berkeley" does not belong to that set. But I cannot be sure whether Clarke's "Observations in Africa" belongs to the set or not. It is for this reason that the set is open. Once the index is made available, I can see if that title is enumerated, and if it is not, I will know definitely that it does not belong to the set of books, which will then become a closed set.

set, and that everything else is not. In the case of an C, we can say that certain things belong to the set, but this is not exhaustive. As we cannot exhaust the elements of the C, we cannot go on to say that everything else is excluded from the set. It may be (and usually is[5]) possible to enumerate some things which are not in the set, but ordinarily this list is also non-exhaustive.[6] And there may be things which cannot be said either to belong or not to belong to the C.[7]

Now, having made this distinction, our antinomies disappear. If we are concerned with O's, there is no O-of-all-O's, but only an C-of-all O's. Obviously there can be no question as to its including itself, as it is of a different nature from what it includes. If we are concerned with C's, then we have no longer the dilemma of an C containing itself or not containing itself. An C may be proved to contain itself, or it may be proved not to contain itself, or it may be indeterminate. Here we must carefully avoid recurring to our difficulties. For, if we make three divisions: the C-of-all-C's-which-include-themselves, the C-of-all-C's-which-do-not-include-themselves, and the residue C-of-those-C's-which-may-or-may-not-include-themselves, and consider that this situation forms a trilemma such that any C must belong to one and only one of these three divisions, we would still have our old antinomy. But this is not the case. An C belonging to the third category is not thereby excluded from the first or second.

A number of the implications of this view are interesting to consider. In the first place, we must make a dis-

[5] But not always. The set of everything is an C such that nothing can be named which is not in the set.

[6] But not always again. The C of all things except the number zero is an exception; and similarly the inverse-set of any O is an C of this type.

[7] The C of all things which are not included in this C is the extreme case. Anything may be said to belong or not to belong to the set.

tinction between infinity as a number, and infinity as a process. Using ∞ as a generic symbol for infinite numbers, both cardinal and ordinal, we will adopt the symbol α for infinite processes. The concept ∞ is philosophically illegitimate, although it is quite satisfactory for mathematicians to use either it itself or any of the transfinite cardinal or ordinal numbers which it represents. Such numbers are "imaginary," and their use is analogous to that of i, the square root of —1.[8]

In addition to using the present system of transfinite mathematics, we may erect a new one by using exclusively the concept α. In this system we will find that some of the old transfinite theorems hold, but that others break down. Among those that break down, as we have shown before, are those concerning the antinomies of infinity. The system explains a fact which is apparent, but unexplained, in the usual system: namely, that infinities cannot be subtracted from or divided by other infinities. If from α (a cardinal process or series), we subtract n, a new process results, such as 0-n, 1-n, 2-n, 3-n, - - - - -, which is identical with the original series. Similarly in the case of $\alpha \div n$. We are dealing, that is, with a directed variable, which can be compounded by any arithmetical method with one or more constants; a new directed variable resulting. But two directed variables cannot be compounded to produce a definite directed variable. Each variable being independent, the order of compounding remains indeterminate and any number of different directed variables may result from various methods of compounding. A special case arises where all methods of compounding two or more directed variables produce new

[8] The analogy may be extended by making an antinomy out of $\sqrt{-1}$. $i \times i = -1$. If i is —1, it must be +1, and if it is +1, it must be —1. And if the $\sqrt{-1}$ is analogous to the set-of-all-sets-which-do-not-include-themselves, the $\sqrt{+1}$ is analogous to the set-of-all-sets-which-include-themselves.

directed variables which have a quality in common. Thus, in the special case of addition and multiplication of two α's of the same order, the various indeterminate resulting variables are all of the same type as the original α, and it may be said that $\alpha_n + \alpha_n = \alpha_n$, or $a_o \times a_o = a_o$.

A curious aspect of this system is that Cantor's proof that $2^n > n$ breaks down. $2^{\alpha(a_o)}$ is the process 2^0, 2^1, 2^2, etc., which is α (a_o). That is, $2^{a_o} = a_o$, when a_o is a process, not a number. Consequently there is no mathematical method of producing an infinity of a higher order than a given one, and the antinomies of cardinal infinity do not occur. All orders of infinity must be generated by some method other than mathematical, and as a result it seems likely that there are not more than two or three orders of infinity: a_o, the infinity of a discrete series; a_1, the infinity of a continuum; and possibly a_2, the infinity of an infinite collection which cannot be ordered.

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THE PROBLEM OF INCOMPATIBILITY IN THE PHILOSOPHY OF ORGANISM

THERE is one question that must have troubled many of Prof. Whitehead's readers in the last few years: the question of metaphysics. I say, troubled; because for most of them it had been answered and done with long ago.

For, ever since it first occurred to John Locke, meeting with a group of friends in one of those genial discussions where the problems of the universe are settled over the tea-cups, that "before we set ourselves upon inquiries of that nature, it was necessary to examine our own abilities, and see what objects our understandings were or were not fitted to deal with,"[1] the conviction has grown that, not the world at large, but the mind of man is the proper subject for philosophical study. Sufficient witness thereto were the titles of some of the main philosophical works of the seventeenth and eighteenth centuries: Locke's own, *Essay Concerning the Human Understanding;* Hume's, *Treatise on Human Nature,* and *An Enquiry Concerning Human Understanding;* and Kant's three Critiques: which would show plainly enough that the concern of these thinkers was no longer to explore the universe in terms of a system of concepts, but rather to examine the adventurous reason itself, define its capacities, and expose its limitations. There were exceptions; but, of these, Leibniz was a voice crying in the wilderness; and the logical romances of the nineteenth-century idealists were always taken with a grain

[1] *Essay Concerning Human Understanding,* Epistle to the Reader, p. 9. Mary W. Calkins ed.

of salt, admired as works of imagination rather than credited as so much information about the nature of things.

And now there comes a man who is neither a romantic poet nor a religious apologist; who would hardly be deemed deficient in scientific sophistication (being himself conversant with mathematics and physics, and a mathematical logician of no mean repute); and who still proclaims himself the champion of "speculative philosophy." Common thought, even common language, he tells us, is instinct with metaphysics; common sense being itself a species of metaphysics, not least ambitious for keeping to the dark. The question is not whether one shall speculate or not; it is rather how one shall speculate, and whether one shall use tacit and dogmatic, or acknowledged and criticized, assumptions. In proof whereof he gives us a cosmology of his own, an endeavor "to frame a coherent, logical, necessary system of general ideas in terms of which every element of our experience can be interpreted."[2] Nor is this done in defiance of the ego-centric predicament which imprisoned Locke, Hume and Kant in epistemology: unlike the neo-realists who cheerfully cut the Gordian knot, Prof. Whitehead patiently unties it; accepting the subjectivist principle, "that the whole universe consists of elements disclosed in the analysis of the experiences of subjects,"[3] he frames his metaphysics in those very terms which enable him to cope with the theory of knowledge.

Is he right? I do not propose to answer that question. My aim is less ambitious: to take my reader with me a little way in a pragmatic—pragmatic in method, not in creed—enquiry, which would judge the system in question not on any preconceived standards, nor even by Prof

[2] *Process and Reality*, p. 4.
[3] *Ibid.*, p. 252.

Whitehead's own criteria of coherence, logical force, and necessity (since to do either would be to beg the question from the start), but by observing the system in action. I start with a problem. I go to his philosophy, and ask how much light it throws on my problem. By its fruits I shall know it.

1. *The Problem: Incompatibility*

Although the problem has figured prominently in recent discussions of foundational logic,[4] no explanation of it has yet been given in terms of the familiar "atomic," non-symbolic logic. I may well begin my account by touching on this phase of the subject.

Consider for a moment the following propositions:

1. Socrates has long, black hair.
2. Socrates has short, black hair.
3. Socrates has short hair.
4. Socrates has long, white hair.
5. Socrates has gray hair.
6. Socrates is bald.
7. Socrates is this man, (as he might have been pointed out by a contemporary Athenian boot-black).
8. Socrates is a town in Kansas.

The observed incompatibilities in these propositions throw some light on the meaning of the term:

A. No two of these propositions are necessarily incompatible unless we assume that they refer to the same subject, Socrates. Identity of subject is, therefore, presupposed in the assertion of incompatibility.

B. No two are incompatible unless they fall within the same universe of discourse. Propositions 1 and 8 are not properly incompatible; they are irrelevant to each

[4] *Cf.* H. M. Sheffer, *Trans. Americ. Math. Society*, vol. xiv, pp. 481-8, especially the last three pages; Whitehead and Russell, *Principia Mathematica*, 2d ed., new introduction to vol. I; and Bertrand Russell, *Introduction to Mathematical Philosophy*, ch. xv.

other. To say that a town in Kansas has long, black hair, is neither true nor false: it does not make sense.

C. Two propositions, with a common subject and in a common universe of discourse, are incompatible, when they present different "determinates" under the same "determinable."[5] Take 1 and 2: their common determinable, "having black hair of any length" has different values, "long" for 1, "short" for 2. In the same way 1, 4, and 5 agree in the determinable "having hair of any color," but disagree in presenting the incompatible determinates "black," "white," and "gray." So 1 and 6: the determinable is "having hair," with positive and negative determinations.

All determinable things are singly determinate.

x is a determinable thing.

Therefore, x is singly determinate.

The middle term "determinable things" could be further explained as, "subjects whose predicates represent one species of quality under a distinct genus"; or, in less traditional terms, as "subjects whose predicates are particular values of a given function." But the minor term, "singly determinate," can only be discussed by leaving deductive for inductive or empirical logic. Here we find that it can be analysed into:

(1) Perspective of determination. Strict incompatibility between propositions with diverse determinates (i.e. 1, 2, 5, and 6) implies that they all refer to the same time and place. It is quite possible that Socrates had long, black hair at one time, short, black hair after a visit to the barber the next week, gray hair twenty years after, and none at all at the time of his trial; and at a given time his hair may have been long and black in one part of his head,

[5] The terms are borrowed from W. E. Johnson, *Logic*, vol. I, chaps. xi and xii; I am not sure, however, that he would agree to the wider sense in which I use them.

short and black in another, and gray about the temples. To state the date and place of the determination in terms of the perspective of one observer so that it shall be translatable into the perspectives of other observers, may seem simple enough in this case; but elsewhere—i.e. in modern physics—it can only be done through the complicated formulas of the theory of relativity.

(2). Standard of measurement. "Long" and "short" are relative terms. A haircut that may have seemed indecently short to the esthetic Athenian, may offend by its length the hygienic sense of the modern American. Nor is this evaded by substituting feet and inches for the vague terms "long" and "short." On the contrary, numerical measurement brings out, that all measurements are made with regard to certain standards fixed by convention, and assumed to remain unaffected by the lapse of time or by their transportation in space.

The moral of this seems to be the relativity of incompatibility:

It seems possible to speak of a "more or less" of incompatibility. Every determinable defines a continuum of possible determinates. There is an infinite number of shades of color, one hue passing into another through an infinity of intermediaries of apparently imperceptible difference. So number defines a continuum of natural, rational and real numbers; so sound, shape, etc. How great need be the difference between two shades of color, or between two decimals, before we can pronounce them incompatible? How much hair must a man lack in order to qualify as bald? The question is an old one: the perception of infinitesimals. Hume had at least the courage to acknowledge it, if not the wisdom to answer it. (*Cf. Treatise*, Part iv, Sec. vi, pp. 537-9, Greene and Grose ed.)

It seems necessary to specify the context in which incompatibility is asserted. "x is a circle" and "x is a square"—are these two propositions incompatible? The question is unanswerable unless we specify in what context those figures appear. If the context is that provided by the postulates of Euclidean geometry, then we can positively assert their incompatibility. Otherwise, it is only a matter of ingenuity to construct a set of postulates where a round square is a commonplace. Paradoxical as it may seem, this further relativity of incompatibility to context seems the best suggestion out of the difficulties of the former relativity of the "more or less." Can we safely affirm that two observations, which agree to the third decimal, are not incompatible? "For all practical purposes" we can. For purposes of exact determination we cannot.

Such specialized uniqueness of determination—the banishing of the "more or less" of incompatibility through narrowing down the point of reference with respect to which that incompatibility holds—is well illustrated in art. A work of art consists of the subordination of details to a pattern, so that elements quite compatible in other contexts become mutually irreconcilable if severally inimical or indifferent to the specific aim of the artist. Take, for instance, the words "sidewalk" and "pavement," words with exactly the same meaning, and almost the same rhythm. Try to substitute "sidewalk" for "pavement" in the following lines of Yeats—where the context is one of powerful harmonies of vowel-values—and the result is disastrous:

> "I will arise, and go now, for always night and day
> I hear lake-waters lapping with low sounds by the shore;

While I stand on the roadway, or on the *pave-*
ments gray,
I hear it in the deep heart's core."

In the same way the perfection of the proportions of a great building like the Parthenon consists in the delicacy of workmanship, that pursues excellences so minute—as in the entasis of the columns or the curvature of the stylobate—as to be imperceptible to the eye, yet marvellously to contribute to the felt rhythm of the whole.

Germane to this is the question of ethical self-consistency, or the incompatibility of given acts with professed principles and pursued ends. An object of desire, like any determinable, defines a continuum of determinates: such and such a "sort of thing" is wished for; and this, subject, of course, to the limitations of the powers of the agent and the fertility of the environment, may be realized in an infinity of actual situations, each achieving the projected satisfaction, though each in a slightly different context. For instance, I desire a drink of water. I may get my drink in a receptacle of any sort: glass, cup, or jug of an infinity of possible shapes, makes, colors, etc.; or I may drink it, a la Turca, in my hand from a faucet, again with innumerable possible faucets; or I might even conceivably lap it up, like Gideon, from a running stream, and do this again in an infinity of alternative ways. In all these cases the desired end would be perfectly, though variously, attained. The question of the "more or less" would enter in when,

1. Environmental limitations were such that water we unavailable, and I should have to quench my thirst with a substitute: an orange, tea, milk, or—ὅ μὴ γένοιτο—wine.

2. My initial desideratum were not simply the function of a single interest, but represented the intersection of more interests, i.e. esthetic, personal, or ritualistic: this

sort of drink, from the hands of this sort of a person, under this sort of conditions.

In either case an infinity of possible consequences would arise, each more or less incompatible with the given end. The only way of ultimately pronouncing upon moral incompatibilities, would be on the basis of a critique of ends or standard of values, which, in unreflective behavior, instinct so quietly and efficiently provides.

For all its sketchiness, this discussion must have made at least one thing clear: the nature of a "non-metaphysical" treatment of a general concept. The maxim of such a treatment is—strangely enough—Platonic: "To keep on the safe side, one must be most on his guard against resemblances: they are slippery old things!"[6] Its ideal is to treat the problem separately for each of the fields wherein it occurs—deductive or inductive logic, esthetics, ethics, etc.—and ascertain its precise import in each case in concepts primarily applicable to that particular field, and not necessarily meaningful in any other. The actual fact that certain terms carry over from one field into another seems, for this point of view, a matter of no great consequence, due to the vagueness of language, rather than to any intrinsic equivalence of the problems involved. To unify these different kinds of incompatibility, as they meet us in separate fields of experience, under a single concept, and to explain that general concept itself by exhibiting it as an element in an even larger system, this "non-metaphysical" standpoint would regard as neither feasible nor opportune. It is to just such an attempt to understand the problem of incompatibility by determining its place in the wider scheme of experience as a whole, that I now turn.

[6] *Sophist*, 231a.

2. *The System: The Philosophy of Organism*

A. The One and the Many.

The world consists of many individual elements; but it is one world, because each of these elements is a center of experience, mirroring—with various degrees of relevance—the whole world. Three basic principles will serve as a necessary commentary to this hasty summary:

1. The Ontological Principle: "nothing is to be received into the philosophical scheme which is not discoverable as an element in subjective experience."[7] Let no one be alarmed over the subjectivistic smack of such a statement. It only means: what *is,* is *in* experience, or *an* experience. The world, being, reality, existence, or whatever other term one wishes to use to describe the metaphysical situation, is to be conceived as a complex of experiences, in which no element appears, save as a subject of, or participant in, some experience. Centers of experience, therefore, ("actual entities," "actual occasions," or "actualities") are the fundamental units, in terms of which everything else is to be understood.

2. The Principle of Relativity: "every item of the universe, including all the other actual entities, are constituents in the constitution of any one actual entity."[8] The ontological principle says, everything that is, is actual. The principle of relativity adds, each thing is what it is, because everything else is what it is. It comes as a flat denial of the old notion of substance as "an existent thing which requires nothing but itself in order to exist":

> "every actual entity is in its essence social and requires the society in order to exist. In fact, the society for each entity is the all-inclusive universe."[9]

[7] *Process and Reality,* p. 253.
[8] *Ibid,* p. 224.
[9] *Religion in the Making,* p. 106, and p. 108 (latter abridged).

"Prehension" is the word which the author has coined to express this fact of universal connectedness, a connectedness that can best be understood as the unity of the experienced world in an experiencing subject. This sounds quite close to Kant; and it is—its exact opposite. Kant says: the unity of the world is the product of the perceiving mind. Prof. Whitehead says: perceiving (prehending) minds are the products of the unity of the world.

3. The Principle of Intensive Relevance: "any item of the universe . . . has its own gradation of relevance, as prehended, in the constitution of any one actual entity."[10] So far we have been told of a world, a connected world, a world of many things. Now we are told of a world of many *different* things. For if everything prehended impartially the whole world, and the whole world was nothing but the sum-total of these prehensions, then all prehensions, uniform summaries of the same world, would be exactly alike. "Relevance" introduces the differentiating principle. Prehensions are different, because they prehend differently: while every item in the universe figures in each prehension, it figures much more significantly in some than in others: all are invited to the feast of prehension, but the seats of honor are reserved for a few.

B. Change, Permanence, and Value.

We start with change: "that which is always becoming, and never really is"[11]: the flux, process, the passage of nature, "the creativity whereby the actual world has its character of temporal passage to novelty."[12] It is the other side, "that which always is and never becomes,"[11] that needs explanation. Recurrence is a half-way house between permanence and change. This is expressed by the universals, "forms of definiteness," "ideal entities," or

[10] *Process and Reality*, p. 224.
[11] *Timaeus*, 27d.
[12] *Religion in the Making*, p. 90.

"eternal objects." But these, like everything else, are subject to the Ontological Principle: they have no being apart from events whose character they define. If so, how can they furnish the basis of permanence? The answer is a kind of Augustinian Platonism: forms are eternally relevant to the passing world, by virtue of their inclusion in the "primordial appetition of God." God is the first-born of the creative process: the original envisagement of all eternal objects as possibilities for realization within the temporal world.

The two problems—the one and the many, permanence and change—meet in the concept of "concretion" or "concresence," "the process in which the universe of many things acquires an individual unity in a determinate relegation of each item of the 'many' to its subordination in the constitution of the novel 'one.' "[13] How does the new individual come into being? By a certain kind of prehension, with a specific kind of relevance. Relevance of what, and to what? Of the whole universe to the purpose, or "subjective aim" of that entity. For every actual entity, in the beginning, before it is anything else, is a subjective aim, an "appetition," an ideal, a desire restlessly seeking satisfaction. This is the selective unit, the principle of novelty. It operates upon the environment, which is that part of the creative process which has preceded it, and which is now the "datum" "objectified" for the subjective aim, the raw material in which this as yet formless will may achieve a concrete realization. This is value: the satisfaction of interest. And the measure of value is the fulness, or "width" and "intensity," or concreteness of the satisfaction.

3. *The Problem in the System*

And now the answer to my problem: Compatibility, in

[13] *Process and Reality*, p. 321.

the generic sense, means togetherness of feelings in an actual occasion. First, an application of the Ontological Principle: "If some group of items . . . can coexist in one actual entity, then the group . . . is a compatible group."[14] Then, a specification of the elements which may thus coexist, in which the author might have well referred to the etymology of in-*com*-*pati*-bility: " 'Feelings' are the entities which are primarily 'compatible' or 'incompatible.' "[15] Incompatibility appears in the genetic process of concrescence. It figures here in two ways:

1. Within the subjective experience of the concrescent entity, it safeguards the unity of the projected aim
2. In the "objective immortality" of the achieved satisfaction, it fixes the character of its contribution to the concrescent process as a whole.

Let me explain:

1. Incompatibility in the Self-Realization of an Actual Entity.—An actual entity is like a cell, "appropriating, for the foundation of its own existence, the various elements of the universe out of which it arises."[16] It is an organism with needs—its subjective aim. Its life—the concrescence—is a series of phases in which those needs are progressively fulfilled, by continuous adaptation to its environment. That adaptation is necessarily selective: its "initial datum" is the selective field; its "subjective aim" is the selective standard; the "feelings" or "prehensions" are the selective acts. Those elements in the environment which cannot be together selected as jointly subservient to the subjective aim are branded incompatible.

A paradox arises here: Unlike the biological organism of the preceding paragraph, an actual entity in its inception is not a full-grown agent, does not yet exist, is not yet

[14] *Process and Reality*, p. 225.
[15] *Ibid.*, p. 225.
[16] *Ibid.*, p. 335.

actual at all, but only a candidate for actuality, only a wish, a hope, a dream seeking realization, with its purpose as its only foothold upon existence. How then can we speak of selection (or "feeling," which amounts to the same thing), as if we had a mature subject capable of adaptive action towards envisaged ideals? The First Category of Explanation, the Category of Subjective Unity, is our author's way out of the impasse: in which we are given to understand that the projected satisfaction—which is the aboriginal *ratio essendi* of the entity—is already operative from the very beginning of the concrescence, eschewing incompatible elements that would lead to feelings ultimately disloyal to the transcendent end.

2. Incompatibility in the Determinate Character of the Actual Entity.—If incompatibility has a very definite ego-centric significance, its meaning is also objective. The concrescence of the actual entity once completed, it has ceased to be a subject, it has now become an object; and an object, we are told, is "its capacity for being a realized determinant of process."[17] The mature satisfaction can now look forward to an "objective immortality": its definite contribution to the growing universe, its specific influence on every future concrescence. Incompatibility here spells the uniqueness of the rôle which that satisfaction must play in the ensuing episodes of the concrescing universe. No matter in what satisfactions it is incorporated, no matter how many times it is cast and recast, it always retains its modicum of stubborn individuality, its ultimate incompatibility with all that is not itself. Such an interpretation of the "laws" of Self-Identity, Non-Contradiction, and Excluded Middle is what we are given in the Second and Third Categories of Explanation. They state the "grounds" of incompatibility: that a complete satisfaction

[17] *Ibid.*, p. 336.

has a character which, good or bad, is at least definite beyond "the shadow of a doubt"; definite by reason of what it is not, as well as by virtue of what it is;[18] and that this definiteness, once actual, remains inviolate forevermore.

Incompatibility, thus generically defined, may be further explained in terms of two supplementary concepts: pattern and contrast. They both refer to the specific context—ideal or actual—in which two elements may or may not be incompatible. Pattern is the "manner" or mode of togetherness. Contrast is the realization of a pattern in experience. Richness of concreteness (which measures depth of value and intensity of satisfaction) depends on the harmonious diversity of various elements held together in feeling through a complex pattern. Incompatibility, therefore, in whatever guise it may appear—logical contradiction, scientific ignorabilia, esthetic discord, ethical conflict—is by no means an unmixed evil. For it may be transcended in a more complex pattern; then a richer harmony springs from the initial confusion: heightened subtlety and rigor of distinction in logic, greater precision and wider generality in scientific law, the larger synthesis in art, the instructed conscience, chastened desire, steeled will in moral endeavor. All of which, a pragmatist and an absolute idealist might hear with approval, and pronounce in chorus: "Thou art not far from the kingdom of heaven."

It remains to translate my previous, "non-metaphysical," discussion of incompatibility into the categories of this system:

Logical incompatibility, I said, is the relation which subsists between two propositions, in the same universe of discourse, when they present an identical subject with

[18] "The fact of incompatible alternatives is the ultimate fact in virtue of which there is definite character." *Process and Reality*, p. 367.

different (incompatible) determinates under the same determinable. We first learn what a proposition is: a tale "that perhaps might be told about particular actualities,"[19] the definite possibility that a certain form might qualify a given existent. Truth, therefore, is a purely adventitious property of a proposition,[20] a dignity which accrues to it only when entertained by a curious, anxious, adventurous being, who is the only one properly concerned with incompatibilities. The subject of a proposition is an actual entity, not in its full concreteness, to be sure, but an actual "it" which can "supply the element of giveness requisite for truth and falsehood."[21] As for the universe of discourse, the author does not explicitly recognize the concept; yet, I think, it may be correlated to his interesting notion of "cosmic epoch," defined as "that widest society of actual entities whose immediate relevance to ourselves is traceable."[22] For a universe of discourse means no more than this: Is a certain concept applicable to a given subject, so that the joining of those terms in a proposition can be significantly asserted or denied? A cosmic epoch answers just that question: it informs us of that particular structure, peculiar to our epoch and not necessarily applicable beyond it, which makes propositions not in themselves true or false, but qualified candidates for such categories. So the "determinable" and "determinate" of the previous discussion reappear, duly generalized, in the doctrine of the "interconnection between the degrees of relevance of different items in the same actual entity."[23] Incompatibles are those "groups of items" (determinates) which "in certain respective gradations of relevance" (under the same

[19] *Ibid.*, p. 392.

[20] "Its own truth, or its own falsity, is no business of a proposition." *Ibid.*, p. 394-5.

[21] *Ibid.*, p. 395.

[22] *Ibid.*, p. 139.

[23] *Ibid.*, p. 224.

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determinables) "cannot coexist in the constitution of one actual entity."[24] Having once decided, in accordance with the Ontological Principle, that "eternal objects tell no tales about each other,"[25] that is, that apart from actuality forms stand in no determinate relations to each other, the author is hard put to it to explain the "proximate relevance" between determinates such as red, yellow, blue, and the determinable, color. But his Augustinian Platonism comes to the rescue, and he can say that "the proximate relevance" means "relevance as in the primordial mind of God."[26]

Perspective of determination and standard of measurement, into which I analysed the notion of "singly determinate," are discussed in the Theory of Extension.[27] This is considered by many the most brilliant part of Prof. Whitehead's work. Certainly it stands today unrivalled in philosophy as a consistent account of the geometry of spatio-temporal perception. To explain this in any detail were obviously out of place here. Suffice it to remark that space-time in its widest sense is interpreted as that geometrical pattern to which objectifications must conform: that general systematic character of our epoch which provides the how of the transmission of satisfactions. A perspective of determination is then the "extensive quantum" or the "standpoint"[28] or "duration"[29] of a satisfaction; a spatio-temporal slab simultaneous with that percipient, with an absolute determination of date and location within that perspective, and a given formula by which observations can be translated from one perspective to an-

[24] *Ibid.*, p. 225.

[25] I have not been able to locate this in his printed work. It is a frequent saying of his in his classes.

[26] *Op. cit.*, p. 173.

[27] See *Principles of Natural Knowledge*, Part III; *Concept of Nature*, chaps. iii - v; *Process and Reality*, Part IV.

[28] *Process and Reality*, pp. 435 ff.

[29] *Concept of Nature*, p. 53.

other. Congruence, once defined as "a certain definite analogy of function in a systematic complex [of straight lines], which embraces both congruent elements,"[30] the standard of measurement then becomes a convention as to the unit of congruence.

And here I would end the discussion: no surer than when I started, but wiser. My question has not been answered; for it should never have been asked; answer enough to expose it. My query concerned metaphysics, not this or that particular brand of it, but metaphysics as a whole, the general ideal of world-construction, exemplified for me more forcibly in Prof. Whitehead's cosmology than in any of the modern systems—shall I accept or reject it? And the answer was a reproof, gentle but firm: Why accept or reject it? Understand it, if you can; admire it, if you wish; believe it, if you must. But you can neither accept nor reject it. It was never meant to be so taken. It is not a weather-report or a foot-ball score, that it should be either true or false and nothing more.

There is your problem: incompatibility. You need not philosophize about it, unless you wish. If you seek to know what it means, go to your dictionary. There you will find different definitions for it: one for logic, one for inductive science, another for esthetics, still another for ethics. Ask and it shall be given you—according to what you ask: specific, practical answers, for specific, practical questions; general and useless and important answers, for general, useless, important questions. If you only knew why you asked the question, you would know how to take the answer.

[30] *Process and Reality*, p. 508. *Cf.* also *Principles*, chap xii, and *Concept of Nature*, chap. vi.

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THE PROBLEM AND POSTULATES OF EPISTEMOLOGY

1. *The nature and problem of knowledge.*

THE problem of knowledge seems to be the problem of explaining what is *explicable* about knowledge. So for the determination of what is explicable about knowledge, and what not, a proper understanding of the real nature of knowledge seems to be necessary. A thorough enquiry into the nature of knowledge is beyond the scope of this paper. The following observations may, however, be quite sufficient for our present purpose.

It may be observed at the outset that the knowledge we are to take into account in an epistemological enquiry is the concrete knowledge of our normal adult life. What the nature of knowledge is in infants, idiots, people in an unconscious or semiconscious state, etc., seems to be impossible for us to determine with any measure of certainty.

Now in any particular case of knowledge in our adult life, we may distinguish two elements, viz. (a) the subject-object relation, and (b) some judgment of fact. As regards the former element of knowledge, it seems to imply in the first place the independent existence of the object presented to the subject, and secondly the direct and immediate presentation or revelation of the object to the subject. This view of the cognitive relation or the relation between the subject and the object seems to be alone consis-

tent with the deliverance of immediate consciousness. And any view which denies the ultimate character of this relation and its implications may, therefore, be said to run counter to the verdict of common sense. The attempt to deduce the subject-object relation from a non-relational, undifferentiated, homogeneous stuff of experience is, therefore, altogether unwarrantable from this point of view.

The elements of knowledge *distinguished* above should not, however, be held to be *separable* from one another. In point of fact there can hardly be a case of knowledge in which the subject, while knowing the object, does not at the same time know it as *such-and-such*—does not, that is to say, make a judgment. This view of the inseparable relation between cognitive relation and judgment seems to hold good in so far as we mean by knowledge the knowledge of our adult life. Whether it equally holds good of any possible kind of knowledge, we cannot definitely say. In any case, the view of the *theoretical* possibility of a mere cognitive relation apart from judgment does not seem to be logically absurd. But the same cannot be said of the possibility of mere judgment apart from the cognitive relation. Judging necessarily presupposes something to be judged. The material to be judged must be directly and immediately given to the subject that judges.

Now it is a fact of experience that the cognitive judgment, immediately involved in knowledge, may be contradicted by our subsequent experiences. If, for example, we know an object as a round table on a particular occasion, and as an oval table on another, the two cases of knowledge obviously contradict each other. Both of them cannot be held to be true. So the question arises as to how we are to ascertain the truth or falsity of knowledge. If the judgment immediately involved in knowledge had never been contradicted, the question of

the determination of the truth or falsity of knowledge would never have arisen.

The question of the determination of the criterion of the distinction between truth and falsity is too difficult to be solved in the present essay. But it is undeniable that *explicit* judgments of knowledge imply *some* criterion with reference to which such judgments are made. Now these explicit judgments of knowledge may be called *logical judgments* as distinguished from cognitive judgments, immediately involved in knowledge. It may be mentioned here that by logical judgments we mean the judgments of validity which are consciously formed. Cognitive judgments may, therefore, be said to be *logically prior* to logical judgments, understood in this sense.

Now this view of the relation between cognitive and logical judgments does not necessarily mean that cognitive judgments or judgments of fact are absolutely independent of all logical value. That at least some implicit logical value may be involved in the so-called judgments of fact seems to be highly probable. The fact that we are often prompted to act on the immediate occurrence of knowledge, without waiting for subsequent experience to determine its explicit logical value, in the same way as we would have done had our knowledge been previously judged as explicitly true, seems to imply that the so-called judgments of fact have an implicit truth-value or a claim to truth. A detailed discussion of the present problem would carry us beyond the limits of this paper. All that need be observed here, has been already mentioned, viz. that knowledge in the sense of cognitive judgment, involving subject-object relation, is the necessary presupposition of, or is logically prior to, explicit or consciously formed judgments of logical value.

We are now in a position to indicate what the real problem of knowledge is, and what it is not.

In the first place, it may be indicated that if the foregoing account of the nature of knowledge is held to be true, it seems to be justifiable to maintain that any problem concerning the explanation of the relation between the subject and the object, and the implications of this relation, mentioned above, is based upon a misconception about the real nature of knowledge. The principal ground on which we have held to the ultimate and inexplicable character of the cognitive relation and its implication, is, of course, the deliverance of our immediate consciousness. It is indeed true that any argument based merely upon this ground can hardly be regarded as sufficient from the philosophical point of view. So our argument need be supported by the demonstration of the logical absurdity of those theories which treat as explicable what we have regarded here as inexplicable. But this wider task cannot be undertaken at present. It may only be urged here that the position of any theory which is in obvious contradiction with the deliverances of immediate consciousness is considerably weakened. As regards the logical difficulty of traditional epistemological theories which seek to explain the relation between the subject and the object, it may be pointed out that the very possibility of such an explanation seems to presuppose the relation of the object to the subject. It is indeed true that we are immediately aware of the object *as existing independently of the subject*, but its independent existence is something of which we are *aware*. The fact of our awareness of the object is, therefore, as inexplicable as the fact of the independent existence of the object.

Secondly it may be mentioned that though we have no reason to view the possibility of cognitive relation apart

from cognitive judgment to be as theoretically absurd as the possibility of the latter apart from the former, we may regard these two elements of knowledge as inseparable, in so far as the actual concrete knowledge of our normal adult life is concerned—the only kind of knowledge we can possibly consider in Epistemology. And therefore no real problem can arise in regard to the explanation of the relation between them.

We are, therefore, led to hold that the only genuine problem of knowledge is the problem of the explanation of the distinction between the truth and falsity of knowledge. Knowledge in so far as it is viewed to be immediate cognitive judgment, does not involve, as already observed, any explicit logical value, though it may be suspected to involve some implicit truth-value. So the question necessarily arises as regards the determination of the criterion with reference to which the explicit logical value of cognitive judgments, immediately involved in knowledge, may be ascertained.

2. *Epistemology and its postulates.*

The problem of truth and error which we have been led to regard as the only genuine problem arising in connexion with knowledge, may be said to be the proper subject-matter of Epistemology. It is indeed unusual to call a problem epistemological which is generally regarded as logical. We have no objection to calling it logical, but it seems to be more proper to call it epistemological for the following reasons.

In the first place, Logic has not always been understood in the same sense. It has passed through various vicissitudes of career. At first it was a mere enquiry into formal validity; then, a methodology of the inductive sciences; later on it was practically identified with Meta-

physics, and lastly it developed into a highly technical science called Logistic or Mathematical Logic, having absorbed within itself considerable knowledge of mathematics. Secondly, Logic may be held to be possible even by those who do not recognise the *reality* of the distinction between truth and error. Thirdly, those who do recognise the reality of this distinction do not, however, seem to regard Logic as exclusively concerned with the consideration of the problem of truth and error. Logic is with them wider than what is covered by considerations of validity alone. Lastly, if the possibility of a distinct enquiry called Epistemology dealing with the genuine problem of knowledge be recognised, and if it is right to contend, as we do here, that this enquiry in the sense of an examination of cognitive relation and its implications or in the sense of an examination of the relation between cognitive relation and judgment of fact is fictitious, we are led to hold that Epistemology must be possible in the sense of an inquiry aiming at explaining the distinction between truth and falsity of knowledge, and this, as already observed, necessarily demands an explanation.

Now it is not difficult to determine the postulates of the epistemological enquiry as understood in this essay. The problem of validity presupposes, in the first place, the *ultimate character* of the fact of knowledge with all its implications. This postulate can, therefore, be analysed into the following elements: (1) The ultimate fact of the *independent existence* of the object of knowledge; (2) the ultimate and unanalysable fact of the direct and immediate presentation of the object to the subject, and (3) the inexplicable relation of cognitive relation to cognitive judgment—knowledge as fact being understood to be cognitive judgment involving subject-object relation.

The second postulate of the epistemological problem

is the fact of the distinction between truth and error. This distinction must be held to be real in order that Epistemology may be possible. It must be noticed, however, that so far as this postulate is concerned, what is to be postulated is merely the *fact or reality* of the distinction between truth and error, but not its *ultimate or inexplicable character*. If the distinction between truth and error be held to be inexplicable, Epistemology as a scientific enquiry would be altogether impossible. So we see that for the possibility of Epistemology the *explicability* of the distinction between truth and falsity is no less a postulate than the *mere fact* of this distinction.

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CONTRADICTION, LOGIC, AND REALITY

THE aim of the following discussion is to examine the logical principle of contradiction in order to reach some conclusion regarding its applicability as a metaphysical criterion of the nature of reality. The inducement to do so arises from the fact that a very important type of metaphysical theory has accepted that principle as its foundation. Though the immediate purpose is thus limited, a wider question is involved as to the relation between logic in general and a theory of reality. To raise, concerning the principles of logic, any question affecting their validity may seem absurd. But the issue is not necessarily a denial of their validity; rather it is the sense in which they are valid. Two considerations prompt to the raising of this issue. One is that the laws of logic or thought present very great difficulty in their interpretation and formulation; and, if the laws are axiomatic, it is not at all clear why there should be this difficulty. If they are not axiomatic, there then may be reasons for the difficulty, and it may be profitable to enquire into their nature. The second is that it is not clear whether metaphysicians take their stand upon logic or whether they intrude their metaphysics into their intepretations of logic. Mr. Bertrand Russell may have good ground for saying that "Schools should be characterized rather by their logic than by their metaphysic."[1]

The metaphysical theory which makes use of the prin-

[1] *Contemporary British Philosophy*, I, p. 359.

ciple of contradiction as a metaphysical principle is that expounded and defended by Bradley, Bosanquet, Taylor and others. It ascribes a metaphysical significance and priority to logical principles, which it does not ascribe to physical laws, such as that of gravitation. The reason presumably is that it adopts a very special theory of the nature of thought and of its relation to reality, and hence comes to regard the principles of logic as being expressive of the nature of reality, much as contemporary science regards the principles of a certain kind of geometry as being expressive of the nature of space.

"Ultimate reality," it says, "is such that it does not contradict itself; here is an absolute criterion. And it is proved absolute by the fact that, either in endeavouring to deny it, or even in attempting to doubt it, we tacitly assume its validity."[2] "Reality is one in this sense that it has a positive nature exclusive of discord, a nature which must hold throughout everything that is to be real."[3] "If we desire to glance in passing at the metaphysical side of the matter, we may remind ourselves that the real is individual, and the individual is harmonious and self-consistent. It does not fly apart, as it would, if its qualities were internally discrepant."[4] "If we are, in the end, to attach any definite intelligible meaning to the distinction between things as they really are and things as they merely appear to be, we must clearly have some universal criterion or test by which the distinction may be made. This criterion must be, in the first place, infallible; that is, must be such that we cannot doubt its validity without falling into a contradiction in our thought; and in the next, it must be a characteristic belonging to all reality as such, and to nothing else. Thus our

[2] Bradley, *Appearance and Reality*, Bk. II, ch. xiii, pp. 136-7, 2nd ed. rev., 1906.

[3] *Ibid.*, p. 140.

[4] Bradley, *The Principles of Logic*, Bk. I, ch. v., 14, ed. 1883.

criterion must, in the technical language of logic, be the predicate of an exclusive proposition of which reality is the subject; we must be able to say, "Only the real possesses the quality or mark X." We have such a criterion in the principle that "what is real is not self-contradictory, and what is self-contradictory is not real."[5] Freedom from contradiction is a characteristic which belongs to everything that is real and ultimately to nothing else, and we may therefore use it as our test or criterion of reality. "The Law of Contradiction simply confirms and reiterates that assumption of the unity of reality which the Law of Identity involved. Reality, the Law of Contradiction asserts, is a *consistent* unity; which is merely to say over again that it is a unity."[6]

These views are cited at some length simply to give point to the remarks that follow. They raise a number of specific questions. What is the utility of the principle of contradiction as a metaphysical starting-point? What is the relation between that principle and the conception of a whole? How far and in what sense is the principle of contradiction absolute and infallible? Is the principle in its nature and in the range of its validity conditioned by assumptions and ideas of commonsense and of science? Is the principle in some sense after all a principle of *human thought* and expressive of a limitation on the part of human beings and hence not possessed of that significance in relation to the universe which the theory in question supposes that it does possess? Are we justified in imposing *our* limitations upon the universe? These are some of the questions that arise and require to be kept in mind.

The utility of the principle as a criterion and a starting-point for metaphysics is questionable. The statement that

[5] A. E. Taylor, *Elements of Metaphysics*, ch. 2, sect. i.
[6] Bosanquet, *Logic*, II, p. 210.

reality is a unity or is non-contradictory expresses at most merely a very general and abstract condition which all propositions about the nature of reality must satisfy. It provides no clue to these propositions; they must be otherwise found. It comes to have significance and to be of utility when we have different propositions with which to deal. It provides a condition which will rule out some propositions and which will render others tenable. The term "logical," as in a quite old interpretation of it and as in the quotation from Bosanquet, means simply "consistent." But consistency in what? Consistency means adherence to and acceptance of all the implications of some admitted or denied propositions. Ultimately we come back upon the idea that "logical" means adherence to one's definitions and initial propositions, explicitly stated or merely assumed. Logic, however, can say nothing about the validity of these definitions and initial propositions, except in so far as to show that all are not equally primitive or are not all consistent. The principle of contradiction is not a principle from which fresh principles or propositions can be deduced. It is merely a selective principle. It only enables us to decide what we must retain or give up, *if* we accept one or two specific propositions, or what must follow on our definitions or assumptions.

We come to a second question. Is the principle of contradiction a sufficient basis on which it can be asserted that reality is a whole? What is the relation between the principle and the conception of a whole? Does the theory cited above actually argue from the principle to the fact (if it is a fact) that reality is a whole, or is the theory justified in using the principle at all only *if* reality is a whole? Is it the case that the only sound position must be thus stated: if reality is a whole, then the principle of contradiction can be validly used, and only so can it be used? The argument

that follows seeks to maintain that the principle of contradiction rests upon and gives expression to the idea of a whole and consequently cannot be used to establish the proposition that reality is a whole. The latter, if it is to be sustained, must rest on evidence of a nature different from the principle of contradiction. The theory under discussion is, accordingly, in using the principle as a metaphysical principle, assuming the point it claims to establish. The application of the principle to reality is valid only *if* certain conditions are fulfilled; and the principle is not capable of establishing whether these conditions are or are not fulfilled. One of these conditions is that reality is a whole.

This case provides one illustration of the way in which logical principles rest upon some more primitive conception and are seen not to be absolute, and are likewise seen to be possessed of no more infallibility than is possessed by that more primitive conception. There is a sense in which logic is to a great extent an affair of "mind," but it is not in the sense that logic is a science of thinking or reasoning understood as a subjective or psychological phenomenon, or in the sense that logical principles are principles expressive of thought and are prescribed by the latter for the objective world. Nor are the "demands of thought or reason" in any way to be regarded as criteria to which the objective world must conform or as clues to its nature; for they may arise merely out of assumptions never clearly formulated. The sense in which logic is an affair of mind is that it is an affair of meanings or definitions which we formulate or construct and to the implications of which we are tied down. Thus there is a sense in which the compelling force of logic lies in our own act, in our own initial premises or postulates.

Traditional logical theory has concerned itself ex-

clusively with propositions asserting qualities of a whole or asserting the inclusion of a part in a whole. It has taken the idea of whole and part or of whole and attribute as fundamental; and all the theory and its logical principles follow from this initial basis. It has assumed that there are wholes concerning which assertions, and assertions of a certain kind, can be made. In fact, it has implied that assertions can be made only if there are wholes or complexes of qualities. Given that there are wholes understood in a certain sense, that is, to have certain properties and to be capable of certain relations, logical theory can formulate its principles, for the latter are based on the idea of these properties and relations. In this respect logic is in a position similar to every other science. Logical principles rest on an initial postulate. The recognition of this fact makes it clear why so much difficulty has been experienced in giving any definite formulation of logical principles such as Identity and Contradiction, and it also makes it clear in what sense these principles are self-evident, necessary, and certain. They are fundamental principles of logic because they express the nature of the primary conception of logic, namely that of a whole of a certain kind capable of certain relations. They are self-evident, independent, and incapable of proof for the same reason. All the defects, as well as the merits, of logical theory have their source in the initial postulate.

What has been said raises the question whether or in what sense logic, as has been frequently held, possesses a status superior to that of the special sciences and is in a position to prescribe laws to them. The basal conception of logic—that of a whole of a certain kind capable of certain relations—is not something innate or a priori. It is a conception derived from experience by a process of refinement; it is derived from experience of specific concrete

wholes; and the beginning of logical theory historically depended in the first instance upon the elaboration of a conception of a whole in the abstract. The difference between logic and the special sciences is merely one of generality; the special sciences deal with specific concrete wholes or entities, while logic deals with wholes in the abstract and with very general properties of the more specific and concrete relations with which the sciences deal. The validity of logic is accordingly not greater than that of the special sciences. Presumably, if the special sciences gave up the idea that they were dealing with wholes and relations between wholes, logic would find itself compelled to adopt a new basal conception. Presumably also it would be driven to that not directly and immediately but by discovering that its logical principles, based on the conception of a whole, found no significant confirmation in scientific knowledge. Logic would thus seem to be always limited or conditioned by scientific knowledge or by what science says about the universe. Logic, based on a conception that is not fundamental in human knowledge, can hardly claim to be itself authoritative over science or to be fundamental.

The conception of a whole on which logic has been based and which finds its justification in the views of commonsense and to a certain extent in the conceptions of traditional science, is a whole which forms the subject of a proposition and of which qualities are predicated or which is asserted to be a part of a larger whole. The principle of contradiction derives its significance and its applicability from this conception. To interpret the principle and to apply it, a whole is held to have a definite and positive nature; in consequence, only certain qualities can be together qualities of it, while certain other qualities are excluded. The latter qualities, however, in virtue of the assumption about a whole, must be attached to some other

whole or wholes other than that from which they are excluded. This is a condition essential to the intelligibility and applicability of the principle. This is the sense in which the doctrine that all contradiction involves a reference to a whole is valid. But in this sense the principle of contradiction reposes on the empirical fact, established by scientific investigation, that certain qualities are compatible and others incompatible with reference to a specific whole. Logic must simply accept the fact of compatibility and incompatibility as testified by scientific knowledge. Logic has not the ability to decide whether that fact is really ultimate or not. Logic thus reposes upon a scientific view of matter and its properties. What science means by compatibility and incompatibility is for science to say. There is reason to believe that any definition will involve conceptions of time and space. Accordingly, the conception of a whole and hence the principle of contradiction rest upon conceptions of matter and of time and space, and will possess no more absoluteness than do these.

Thus the principle of contradiction is neither absolute nor infallible. It is conditioned by the conception of a whole which is fundamental to logic. It is not infallible, for that conception of a whole presupposes views of matter, time, and space which cannot claim infallibility. It cannot, therefore, be with certainty considered as an absolute and infallible criterion of the nature of reality. Even, however, if it is accepted, it is quite incapable of proving that reality is a whole. For, in the first place, it rests upon the conception of a whole. But, in the second place, the conditions under which it is applicable would render it inapplicable to reality, if reality were a whole. The metaphysical theory which holds that reality is a whole and that the evidence for it is the principle of contradiction falls into confusion regarding the conception of a whole

as the basis of logic. The proposition that the conception of a whole is the basis of logic is not the same as the proposition that there is only one whole and that logic rests upon a conception of *the* whole. The significance and applicability of the principle of contradiction rest upon a conception of a whole and of relations between such wholes. If there is only one whole, the principle in its logical sense is not applicable. In order to be able to apply it and to defend its being applied, it is necessary to interpret the proposition that "all contradiction involves a reference to a whole" as the proposition that "all contradiction takes place within a whole." This proposition does not express the relation which holds in logical theory between a whole and contradiction. Nor is it true, for a whole rejects contradictions, and the metaphysical theory in question makes it do so. Hence, also, it further follows that the argument that all finite wholes are incomplete and contradictory, and because of this incompleteness and contradiction point to an all-inclusive whole, is invalid. The principle of contradiction cannot prove the incompleteness and contradictoriness of finite wholes. Their incompleteness and contradictoriness are asserted on the basis of an *assumption* that there is an all-inclusive whole. Thus at every point it turns out that the metaphysical theory assumes the conception of an all-inclusive whole; and, because in logical theory the idea of a whole and the principle of contradiction are closely connected, it proceeds, though without clearly analyzing that connection, to assert that the principle of contradiction is an absolute principle which expresses the general character of this all-inclusive whole.

Attention has already been drawn to the fact that great difficulty has been experienced in interpreting and formulating in an intelligible manner the principle of contradiction as well as the other laws of thought. There ought not

to be this difficulty if the principle is self-evident. The difficulty is due to the fact that the principle is not self-evident; for it involves ideas that are very obscure and complicated. There has been some controversy as to whether in the formulation of the principle an explicit reference to time is necessary.[7] Whether such a reference should or should not enter into the formulation does not affect the fact that the idea of contradiction involves ideas of time and place. But if so, it is still necessary to understand what is meant; and it is thus necessary to consider the ideas it involves. There is no contradiction in supposing that an object may have two distinct colours at different times. The possibility that two qualities, say two colours, become contradictories lies not in their being different but in sameness of reference, or in their being asserted of or in their claiming to be predicates of the same object at the same time. The idea of contradiction, as has been already noticed, involves the ideas of space and time; and any interpretation of the principle must be bound up with theories about the nature of time and space. This is a point to be borne in mind, because the metaphysical theory under examination, in virtue of the application of the principle of contradiction, reduces space and time to *appearance*. The theory is thus itself inconsistent, for it cuts the ground from under the principle on which it rears its structure.

Another important point must be noticed. The qualities of *different* senses can never possibly become contradictories. A tactile quality, such as hardness, can never become a contradictory to a visual quality, such as red. They can never be incompatible. This means that contradiction rests on the complicated fact that two or more members of the same series cannot occupy the same space at the same

[7] See Ueberweg, *Logic*, pp. 436-7.

time. The possibility of contradiction lies in the nature of the members of the same series; while non-contradiction lies in the nature of the members of different series. This fact has an interesting relevance to the applicability of the principle of contradiction to reality and to the conclusions which may be drawn from such an application. There is an ambiguity in the idea of contradiction and of non-contradiction which has not been generally noticed. That idea has usually been considered as being applicable in one unambiguous sense both to the members of the self-same series and to the members of different series. When contradiction, however, is traced to its roots, the sense in which two visual qualities are non-contradictory is not the sense in which a visual quality and a tactile quality are non-contradictory. In the one case, contradiction means that an object A cannot have the quality red and also the quality blue together, and non-contradiction means that the quality red qualifies the object A while the quality blue qualifies some other object B, two objects being thus involved. In the other case, contradiction cannot occur; and non-contradiction means that an object A can have the quality red and the quality hard together, one object only being involved. The second case is just that where the idea of contradiction has no significance and is not applicable, for there is lacking in it something which renders contradiction possible. The principle of contradiction is thus not of universal validity and capable of universal application, any more than the idea of husband is validly applicable to every human being. It is accordingly not capable of giving a clue to the nature of reality in all its range; and there is no reason, therefore, to suppose that reality is that kind of compact whole which the metaphysical theory of Bradley and Bosanquet, on the basis of the principle of contradiction, maintains it to be.

The question arises as to what is that something which is present in a specific series, for instance of visual qualities, and which constitutes a basis for possible contradiction, while it seems to be lacking between different series and its absence gives another significance to non-contradiction. A consideration of this question leads to an examination of certain ideas which appear in the formulation or else in the interpretation of the principle of contradiction. An object cannot both have a quality X and not have it at the same time, or it cannot have X and non-X at the same time. This means that the principle of contradiction involves the idea of sameness—there must be the *same* object, there must be the *same* time, or there must be *sameness* of reference.

Now it is becoming well-known that these conceptions are full of difficulty; and the fact that there is such difficulty shows quite definitely that the principle of contradiction is far from being axiomatic. The only ground for its being regarded as axiomatic has been that the idea of sameness was thought to be simple and straightforward; and this belief has rested upon certain assumptions regarding space and time. These assumptions are expressed in the conception of the unity of space and the unity of time and in the idea of a public space and a public time. On the basis of such assumptions no difficulty has been felt to be present in the idea of *sameness of place* or in the idea of the *place* where an object is. *Sameness of time,* it has been felt, is a very simple matter to decide; all that is required is to take a reading on a clock or watch and to compare readings on different watches or clocks or by some other means to fix the date of an occurrence. All this is a matter of an age-long science that has become the substantial body of commonsense.

To question the validity of this attitude may seem very foolish. But it is science, the steady destroyer of commonsense, that finds it necessary to question it. For all practical purposes the commonsense view may continue to be satisfactory; but it is incumbent on the philosopher to ask what this means and implies. It is important to do so because these controversial conceptions enter into the meaning of contradiction; and the point at issue is the applicability of the principle of contradiction to the universe or reality. Is there any reason to suppose that what is satisfactory for all practical purposes is also satisfactory for the interpretation of the universe? Is there any justification for limiting the nature of the universe by the requirements of human purposes? What then is implied by *all practical purposes?*

From the purely scientific point of view it means that the difference between the results attained on the basis of these conceptions and the results attained on the basis of more adequate conceptions is so small as to be negligible. But this negligibility is not with regard to the universe but holds only in reference to a part of the universe relatively to which these conceptions remain valid and where the conditions give support to a common point of view. This consideration, however, shows how propositions are relative to a certain point of view. So long as investigators are carrying on their work under similar physical conditions, no factor enters to produce fundamentally different results. The importance of this appears on the side of social life. It is obvious that in social affairs it would be extremely inconvenient if the time, distance, and calculations of the policeman varied from those of a driver of a motor car moving at a great speed.[8] Such a difficulty would not arise

[8] *Cf.* W. P. Montague, "The Einstein Theory and a Possible Alternative," *Philosophical Review*, March, 1924, pp. 152 ff.

so long as rates of speed did not become greater than they are at present; but if they were increasing much, as seems possible, the inherent difficulties of time and space measurements would be acutely revealed. They would reveal that there is no absolute time or space but that the measurements of time and space are merely conventional. Time- and space-measurers are not absolutely neutral; their neutrality is conditioned. The significance of all this is that the conceptions of time and space have been accepted as valid because their application has fitted into the co-operative activities of men, and they have done so for the reason that those who are interested in the matter have been living under physical conditions affecting all alike. Social life itself has acted as a spur to the securing of neutral measurers and common standards wherever necessary in order to effect an harmonious working of communal life. The social factor helps the development and the formulation of the idea of a public space and a public time so that times and places may be the same for all.[9]

Hence the principle of contradiction is not a demand of the reason. Its force does not lie in the rational nature of man. It rests upon the need of securing co-operation in social life. This need is in part satisfied by the fact that physical conditions are such as to facilitate and support a common point of view, and that these conditions reveal no fundamental divergence in men's interpretations of things. The need is also further satisfied by the elaboration of neutral instruments for making and recording observations. The question now, in the light of this, is whether there is any reason to suppose that what is satisfactory from this point of view common to men living under certain conditions and elaborating conventions suitable within

[9] *Cf.* A. E. Taylor, *Elements of Metaphysics*, p. 246, on the social nature of the conceptions of space and time.

the area of their life, is metaphysically adequate to reality. Is there any reason to suppose that their point of view is a point of view valid for the universe? Is there indeed any reason to suppose that, even if it is admitted that the traditional point of view is valid in a limited way, there is possible *one* point of view that is absolute or true for the universe? The belief in the past has been that the point of view of traditional science and of commonsense was absolute and that its implied propositions were valid for the universe; hence it has been held that logical principles, such as that of contradiction, were valid not merely for a specific and human point of view but were valid for reality absolutely.

What requires examination is the foundation for any such belief. To carry this out it is necessary to keep the immediate issue in mind. The metaphysical theory of absolutism asserts that the principle of contradiction is an absolute criterion; it is a criterion of reality absolutely. On the strength of this it proceeds to assert that the propositions formulated with regard to any finite sphere are contradictory, and that there is only *one* point of view for which there is no contradiction. Space and time on this theory are conceptions valid for a limited point of view but for that reason are contradictory. At first sight modern science, by pointing out the inadequacy of the conceptions of space, time, and matter, would seem to give support to this view. But it is only at first sight. Modern science refuses to accept the idea of an absolute point of view. There are points of view, each being defined by certain postulates. The conception of a whole, which has already been shown to lie at the basis of the principle of contradiction and to give significance to it, now appears as *a* point of view. But if more than one point of view is

possible and if there is no one that has a right to be called *the* point of view, two propositions that are seemingly contradictory may yet be true. A contemporary scientist can write: "it can scarcely be said to be a self-contradictory property to be in two places at the same time any more than for an object to be at two times in the same place."[10] The reason is that what are called place and date are dependent on certain factors, such as the axes of reference and the speed of light. The principle of contradiction is thus seen to express the coherency of propositions relative to a point of view. It is a principle of selection and rejection of propositions relative to a point of view; it is not a principle applicable to the relation between two propositions belonging to two different points of view and is not capable of selecting or rejecting either of them, just as it not applicable to any relation between, say, a colour series and a tactile series.

Nor is the principle of contradiction capable of selecting between different points of view and decreeing that one is the right point of view. It is incapable of deciding between the sets of postulates that go to define the various points of view. The scientist has devised formulae for interpreting results attained by reference to one point of view in terms of another point of view. What becomes fundamental is thus a law of transformation, and not the law of contradiction. The idea of co-ordination and mutual compatibility takes the place of contradiction and mutual incompatibility. The co-ordination of points of view becomes the substitute of Bradley's absolute in which all contradictions are *somehow* reconciled. The unity of the universe is not a unity that is capable of being properly expressed in the form of the principle of contradiction,

[10] Eddington, *Space, Time, and Gravitation*, p. 60.

but it is of a purely formal character. There is no absolute point of view in reference to which the nature of the universe can be or is to be interpreted. The metaphysical theory of absolutism is right in emphasizing the limited nature of any point of view and the consequent relative character of propositions or of knowledge, but it is wrong in using this fact as evidence for asserting that there is an absolute point of view. And its paradoxes and its difficulties are due to its making absolute a principle which is itself valid only in reference to a finite point of view.

What the metaphysical theory of absolutism has unwittingly done in accepting the principle of contradiction as a proposition valid throughout all reality is to regard as absolute certain conceptions which are only valid relatively to a special point of view. If its contention is sound, the conception of sameness—sameness of time, sameness of place, sameness of object—must be absolute and have only one ultimate meaning. But there is no such absolute meaning to be attributed to that conception. "A perceptual object," says Dr. Broad, "is really not one single homogeneous object, present in a place in the movement-continuum in one single sense of 'presence.' It is a number of inter-connected objects of different types, and the different kinds of objects included in it are present in different senses in the place where the perceptual object is said to be."[11] Each visual sensum is literally present only at a place in its own visual field; each tactual sensum is literally present only in its own tactual sense-field. When we say that visual sensa and tactual sensa alike are at the same place, we are using "place" in a sense that is defined only by bodily movement. "There is one and only one kind of place which we deal with when once we leave indi-

[11] *Scientific Thought*, p. 330; *cf.* also p. 345.

vidual sensa and their fields and pass to physical objects in the widest sense of the term. This is a place in the continuum of possible positions of our bodies as we move. . . . Wherever we talk of any sensum occupying a place in the movement-continuum we are using terms in a Pickwickian manner and are bound to define them."[12] There is no clear and unambiguous meaning of "place."

There is a corresponding difficulty with regard to the conception of time. Sameness of time is thought to be an easy matter to decide; all that is necessary is to consult a time-measurer such as a clock. Though this procedure is familiar and has become habitual, it rests upon conventions involving complicated conceptions. It rests upon the idea that time is homogeneous and is something distinct from space and matter. It involves the idea of simultaneity and a distinction, necessitated by the fact of the speed of light, between sensible simultaneity and physical simultaneity. Behind it is the traditional view "that there is only one permissible time-direction, which can and must be used for *dating all* physical phenomena."[13] But this view, though of actual utility, need not be any longer accepted. There is nothing that can be called *the* timeless space of nature. More directions than one (though not all) in physical space-time are permissible time-directions; and there is no reason for holding that two events which are simultaneous in one frame of reference will be or must be simultaneous in another frame of reference, or that two events that are in the same place in the timeless space of one frame will be or must be in the same place in the timeless space of another frame.[14]

[12] *Ibid.*, p. 333.

[13] *Op. cit.*, p. 475.

[14] *Ibid.*, pp. 475 ff.

It would seem therefore that the metaphysical theory of absolutism has been assuming that there is an absolute *sameness,* and that it has been applying in the metaphysical sphere that idea of absoluteness which was at one time influential in science. The decay of absolutisms everywhere involves an absolute metaphysics in their fall. The conception of sameness can be given, though there is difficulty and it is not a simple matter, a meaning by reference to a point of view. This is the unity to which the principle of contradiction is supposed to point. The conditions under which the principle is applicable show that it is not a principle applicable to reality. There is no possible way of giving a meaning, from the side of reality in its whole range, to the term "sameness." There is no evidence that reality is an object in the sense in which the principle of contradiction in its ordinary logical usage involves an object; and even if it were such an object, any interpretation would still be subject to the difficulties which have been seen to exist in the case of an object. An examination of the conception of a perceptual object shows that the principle of contradiction comes down to a question of a relation between sensa of a specific series, such as visual sensa or tactile sensa; but then spatial relations in a clear straightforward meaning do not hold between the different sensa of different senses; nor, as has been seen, does the principle of contradiction so hold. The meaning of "sameness" leads back, accordingly, to a problem about sensa and about the way in which they constitute an object and in which an object can be the "same" in different places and at different times.

The metaphysical theory of the whole or of absolutism lays great emphasis upon "unity." The preceding sections represent an attempt to show in what sense "unity" is in-

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volved in the conception of contradiction. The "unity" is a point of view defined by a set of postulates; and within this reference all propositions are coherent and the universe is interpreted so as to obtain a set of propositions that are coherent. But the metaphysical theory asserts and seeks to defend the proposition that the "unity" is a system. Now the preceding examination of the principle of contradiction has shown that the latter is not a sufficient basis for this conception of system. "Systematic" is not synonymous with "non-contradictory." The geometry of Euclid is systematic and non-contradictory. The entities discussed have certain relations to each other; and these relations have logical properties in virtue of which deductions can be made and certain propositions accepted and others rejected. All the propositions hang together in accordance with definitely apprehended and intelligible relations. The emphasis, in the conception of system, is upon the relational factors. A perceptual object will, on this view, fail to comply with the conception of system. There is no clear intelligible relation, possessing certain logical properties, between the visual qualities, the tactile qualities, and so on. Yet there is a sense in which a perceptual object is non-contradictory; it is non-contradictory, as has been seen, because the conception of contradiction is not capable of application to it at all.

The metaphysical theory has maintained the systematic character of reality as a whole in the first sense of system. That would mean that no two constituent propositions of the system could be contradictory. As, in the conception of system, the emphasis is upon the relational factor, to say that A has the relation X to B and that A has the relation Y to C or even the relation X to C would not involve a contradiction; nor would there be a contradiction between

the proposition "A has the relation X to B" and the proposition "A has the relation Y to B." In order to have contradiction, two relations of the same kind must be asserted to exist between A and B, such as two spatial relations or two time relations, so that the relation must be one or the other. Now modern science is prepared to admit that A, a constituent of the universe, may be related to B spatially or temporally in two or more different ways. A in one respect may be before B, in another respect it may be simultaneous with B. There is no reason, therefore, to admit that reality is a system in the sense maintained. Science would not admit that there is *a* proposition which can assert *the* real or true relation. As the conceptions of space and time are so much involved in the conception of contradiction, there may be a deep and subtle motive in the attempt to prove the *apparent* character of space and time. But if space and time are apparent or unreal and thus not to be regarded as inherent in the absolute, the principle of contradiction must also cease to be a principle of the absolute.

There is, however, the second meaning of non-contradictory according to which reality or the universe may be regarded as non-contradictory without being regarded as a rigid logical system. The analogy here is that of a perceptual object which is constituted of a series of sensa of different senses. Just as, in order to give meaning to the term object, the different sense-strands have to be collected by some device or to be correlated, so the universe may be considered as having different strands which by a scientific device are or can be correlated so as to give us some conception of the universe. The visual qualities do not logically imply, as the English empiricists so often pointed out, the other sense-qualities; nor does one strand

of reality need to be considered as logically implying the others. They are not bridged by spatial relations or by any type of relation having logical properties. Relatively to the idea of logical connection a certain amount of looseness or of discontinuity may be characteristic of reality, and there may not be that unity for which the metaphysical theory of absolutism argues.

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AN EXAMINATION OF HUME'S THEORY OF RELATIONS

HUME'S philosophy has been aptly described as "naturalism," inasmuch as the determining factor in his thinking was the attempt to substitute instinct and belief or feeling for reason in accounting for the facts of human experience. One of his chief aims was to show that, except with reference to such relations as those upon which mathematical science is based, belief never rests on rational grounds, that the syntheses of reason are merely generalised beliefs. For instance, the assumption of permanent and identical things is a "natural belief" occasioned by the instincts or propensities of our human nature. The "plain man" takes his perceptions to be the real things; and, accordingly, conceives of them as continuing to exist when unperceived, and as remaining identically the same even though they have undergone change. Yet, in the case of visual perceptions, he has only to close his eyes in order to annihilate these perceptions; and since, on opening his eyes again, the perceptions he has are new perceptions, separated by an interval from the previous ones, to say that simply on account of their resemblance to the latter they are identical with them is manifestly unjustifiable. The identification of perceptions with real things is an "illusion," due to the mind's propensity to feign; but the belief in the existence of external objects thus engendered turns out to be *practically suc-*

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cessful in the ordinary affairs of life, although theoretically it can neither be proved nor disproved. So, again, with reference to the self, the existence of a permanent and identical entity cannot be rationally established. Far from the self being a simple substance, and the most certain existent of which we are cognisant (as the Cartesians held), it evinces itself, from the standpoint of reason, as merely a multiplicity of *discreet* experiences. But we instinctively *believe* in its permanence and identity; and this belief *works successfully* in all matters of practical life. It is, however, as little capable of being theoretically proved as of being theoretically disproved.

It was certainly unfortunate that T. H. Green, in his examination of Hume's philosophy, ignored this doctrine of belief. But that the doctrine is beset with insurmountable difficulties there can be no question. It virtually assumes the very distinction between the conscious subject and its states which it is postulated to avoid, and which would undermine Hume's fundamental position that the sole constituents of experience are isolated perceptions. In trying to discover the nature of belief, he was compelled to suppose that it was either an "impression" or "idea," or a quality of "impressions" and "ideas." The former alternative was precluded, seeing that two persons might have similar ideas in their minds and yet entertain very different beliefs concerning them. It must, therefore, be some quality of "impressions" or "ideas." And the only way in which similar "impressions" or "ideas" can differ from one another qualitatively, is, according to Hume, in respect to the "force" or "vividness" with which they are endowed, so that we may be said to believe a proposition when our ideas are forceful or vivid, and to disbelieve it when they are faint and weak. Now, in the first place, it is impossible to understand the mean-

ing of the terms "force" and "liveliness" as applied to ideas, unless by the term "idea" be meant an "image," which was no doubt what Hume did mean. "Ideas," however, in the sense in which in this context they are relevant, are certainly not "images," and on that account alone the theory cannot stand. And, in the second place, even if we could assign a meaning to the phrase "the vividness of ideas," we should have the hopeless task on our hands of showing how differences in respect of this one quality can be used as the basis of a number of important distinctions. The differences between perception and imagination, between memory and imagination, between belief and doubt are all, according to Hume, due to differences in the liveliness and force of our ideas. Suppose that I (a) remember an isosceles triangle which I drew yesterday, (b) believe that the two angles at the base of an isosceles triangle are equal to one another, (c) imagine an isosceles triangle. How exactly could these processes be distinguished on the view we are considering?[1] In the third place, it is clear that Hume was himself wholly unable to work out this view of belief without calling to his aid, if not in name yet in fact, the mental process which Locke had variously designated as that of combining, distinguishing, abstracting and judging. Now whenever Hume finds it impossible to recognise in an "idea" the mere copy of some original impression, he introduces the qualifying phrase "manner of conceiving." Thus, for example, we have an *idea of necessary connexion* when we assert that one thing is the cause of another. This idea, Hume avers, is the repro-

[1] Similar difficulties present themselves in any theory of belief as a quality of ideas. And any theory which tries to reduce the mind to a 'bundle of perceptions' is forced to such a theory of belief. For example, Mr. Bertrand Russell (in his *Analysis of Mind*) takes the mind to be a complex of sensa and images and interprets belief as a *feeling* which *accompanies* certain of these sensa and images.

duction of an impression which the mind *feels* itself compelled to "conceive in a particular manner." Yes; but it is the *mind*—not the collection of discrete contents which is all, Hume insists, we are justified in taking the mind to be—that conceives, or, in other words discriminates and judges; and, unless the two ideas of a particular cause and a particular effect can be held together in one act of apprehension and thus be thought of in connexion, it would be simply inexplicable how this "manner of conceiving" could come about. Now, it may be said in Hume's defence that he admits the inexplicability. In the present context, that is hardly, I think, true. In any case, however, he does allow that we *have* the idea of necessary connexion—an idea, that is to say, which, whether "fictitious" or not, involves the apprehension of two terms in their togetherness and mutual relation. How, then, can it possibly be maintained that we certainly know of *all* "perceptions" that they are "separate and distinct," when, even from the point of view of empirical observation, we certainly know that *some* ideas are not.

So far as the distinction between "natural" and "philosophical" relations is concerned, Hume includes under the former the three relations of resemblance, contiguity and causation, while under the latter he includes the seven relations of resemblance, identity, spatial and temporal relations, relations of quantity and number, relations of degree, contrariety, and causation. Obviously these cannot be regarded as two divisions of the same class, because, as Meinong remarks, the genus common to both is wanting. Nor can the way in which these two kinds of relation (if one may use this inexact expression) stand to one another be that of mutual exclusion, seeing that, for example, causation belongs to both lists, contiguity and

resemblance would seem as a matter of fact to be both philosophical and natural relations, though not necessarily to subsist together, while there are some philosophical relations (such as distance[2]) which seem to preclude the presence of natural ones. In the long run, it is, however, only "natural" relations that Hume will admit to be true relations. And with these he couples the view that, by virtue of variations in them, the "imagination" (which is, as Pringle-Pattison observes, a "hardly veiled reintroduction" of the conscious self) may determine to group ideas together in particular ways. Association of ideas is, then, the substitute that Hume has to offer for the synthesis that would appear to be essential to knowledge, and it is by the principles of association that he endeavors to explain what are otherwise called judgments of relation. The attempt turns out to be a failure, as Hume himself candidly acknowledges in the Appendix to the *Treatise*. The only combination which, by any show of plausibility, could be accounted for in this manner would be the formation of complex ideas out of simple constituents. But the idea of a relation between two known contents is not rightly to be described as a complex idea;[3] and, as there is no impression from which it could have been derived, Hume was finally obliged to recognise that the association theory of its origin breaks down. Throughout his attempt to employ the theory in this context one can discern a perpetual see-saw between two conflicting methods. On the one hand, association is treated by him as that which gives rise to ideas of relation, and, on the

[2] "In a common way," Hume remarks, "we say that 'nothing can be more distant than such or such things from each other'."

[3] Inconsistently enough Locke had described 'ideas of relation' as complex, and he did so because they involved, in each case, a plurality of compared ideas. But this is obviously no reason for supposing that the 'idea of relation' is itself complex; and, indeed, at other times, Locke speaks of such 'ideas of relation' as succession and power as simple.

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other hand, ideas of relation are treated by him as the features that give rise to association.

The inherent contradictions involved in Hume's treatment will be best brought out by considering the way in which he handles certain of the relations that he enumerates.

(a) The relation of *resemblance* is said to be necessary to relation of any kind, although, as Meinong urges, it would be hard to show how it is involved in such a relation as that of cause and effect. But the fundamental difficulty in Hume's account of resemblance is this. He informs us, on the one hand, that such a perceived object as a "globe" contains many different resemblances and relations, and, on the other hand, that it consists in "the impression of a white colour disposed in a certain form." Now, however gladly one would accept from Hume acknowledgment of the fact that the content perceived is not, in such a case at least, a simple unit, however readily one would recognise his acknowledgment that "resemblances and relations" are comprised within this content, yet the acknowledgment must be pronounced to be wholly irreconcilable with his general view. If, in the case cited, the "globe" is an "impression," and if "relations" are "ideas," how *can* the former contain "different resemblances and relations?" An *idea* of resemblance can, as Hume is reluctantly compelled to admit, only arise through an act of comparison, by which various "impressions" are found to be partly identical and partly different. If, then, we have the "impression" of a "globe," that must imply that we have previously gone through a process in which various objects have been perceived and discovered to resemble one another, which means, in other words, that the "idea of resemblance," so far from being just a

"related idea," is the product of an act of judgment. An idea qualified by relation of resemblance to other ideas is, in short, a very different thing from an idea of resemblance—different, as Green puts it, with all the difference which Hume ignores between sense and thought. If it be urged that, in the case of the complex content just referred to, we first apprehend directly its various features, and *afterwards* discern that these resemble the features of other complex contents we have had before us, then the reply is that by such a contention the difficulty is only thrown a stage further back. If the content now before my mind has various features, these must be distinguished from one another, and the act of discriminating is identical in character with that by which one content is found to resemble another.

(b) Following a distinction which Hume himself does not make, but which is now sufficiently familiar, we may speak of the formal and real relations involved in conscious experience. Under the former are included spatial and temporal relations, which are universal elements in our experience. The discussion of them is opened by Hume with the consideration of a specific problem, the problem, namely, as to whether space and time are, as the mathematicians supposed, infinitely divisible. In conformity with his general view, Hume was bound to maintain that they are not, that such infinite divisibility is a fiction. He persists in regarding the ultimate elements of experience as discrete units, capable of being represented in isolation as ideas. In what orders or classes of impressions are, then, the units of space and time to be found? Hume's answer, when freed from much ambiguous phraseology, is briefly this. Certain impressions, those, namely, of sight and touch, have in themselves the characteristic of extension, because these impressions (in

this context Hume speaks of them as "points") exhibit a certain order or mode of arrangement—which order or mode of arrangement is common to both coloured points and tangible points and which considered separately is the impression from which our idea of space is taken. And so, too, in respect of time. All our impressions and ideas are received in a certain order, the order of succession. This *order* is itself the impression from which the idea of time is derived. "The ideas of space and time are no separate or distinct ideas, but merely those of the manner or order in which objects exist."

After what has been already said, it is hardly necessary to emphasize the two-fold inconsistency here—first, in describing the *order* in which the impressions are arranged as itself an impression, and, secondly, in describing *coexistence* and *succession* as ingredients in an experience which actually consists of isolated units. On the one hand, the "points" which are said to be in a certain order must be *ex hypothesi* themselves impressions. If, then, *any* impression can be said to be one of "coloured (or tangible) points disposed in a certain manner," this can only mean that the impression in question is or consists of such "points." And, indeed, in one passage, Hume expressly describes extension as "a compound impression, consisting of parts or lesser impressions, that are indivisible to the eye or feeling, and may be called impressions of atoms or corpuscles, endowed with colour and solidity." Yet, unless these "lesser impressions" can be regarded as present together, they must follow one another, and thus precede the "compound impression." On the latter supposition, extension would consist of parts none of which could be present at the same time, and all of which must cease to be present before extension itself could come into being. But the former supposi-

tion is precluded by what Hume inculcates in regard to time. It is true he does not assert in so many words that all visual impressions must be successive, but he *does* assert that "the impressions of touch," which along with those of sight he had represented as constituting the "compound impression" of extension, "change every moment upon us." And after having made out extension to be a compound of coexistent impressions, he proceeds to speak of the idea of time as derived "from the succession of our perceptions *of every kind*, ideas as well as impressions." The parts of time cannot, he urges, be coexistent; and seeing that "time itself is nothing but different ideas and impressions succeeding each other," it would follow that "the parts of time" are those "perceptions of every kind" from which the idea of time is derived. If, then, *all* impressions, as parts of time, are successive, how can some impressions, as parts of space, be coexistent? On the other hand, not only does Hume fail to account for the awareness of coexistence, he equally fails to account for the awareness of succession. The bare perception of impression after impression, even supposing such perception were possible, would not of itself constitute such awareness. A succession of perceptions is one thing, a perception of succession is quite another, and from the former to the latter there is "no road." How could a sequence of impressions of which no two are present together, undetermined by relation to anything other than the impressions themselves, yield a consciousness of the relation between the moments in which the impressions are given, or of the sum of such moments? As Kant was presently to insist, no apprehension either of coexistence or succession is possible save in reference to a permanent over against which temporal changes can be recognised.

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From his view of space and time there follows in Hume's *Treatise* the most extreme empiricism with respect to the nature of mathematical reasoning—an empiricism which does not, however, extend to the propositions of arithmetical and algebraical science. With respect to these, he apparently held that, since each element of conscious experience is presented as a unit, and since we are capable of considering any complication of facts as a unit, our *manner of conceiving* is absolutely general and distinct, and that upon it there could be based a perfectly accurate and general science, that, namely, of discrete quantities. But in regard to geometrical science, since the data are facts of experience apprehended in a wholly empirical fashion, the results, which rest on comparisons themselves empirical, can never be more than approximates. Geometrical propositions imply an exactitude which does not correspond to actual knowledge. "Though it (i.e., geometry) much excels, both in universality and exactness, the loose judgments of the senses and imagination, yet it never attains a perfect precision and exactness."

Hume's account of mathematical relations, full, as it is, of obscurity and of irreconcilable statements, is in itself a sufficient refutation of his whole theory. From the well-known passage in the *Inquiry*, where the distinction is drawn between relations of ideas and matters of fact (§ iv. pt. 1), one would gather, as Kant did, that, according to him, mathematical propositions are all of them analytical, true on the ground that denial of them would amount to a violation of the law of contradiction. In reasoning on mathematical data, he appears to say, we may proceed by a mere contemplation of our ideas, because the ideas exactly correspond to the impressions, and mathematical propositions have, therefore, a truth

or falsity, independent of the concrete existence of the objects which may be implied in them. But, seeing that no more can be got from the "ideas" than is contained in the original impressions of which they are "copies," a *relation* between ideas must *either* be an actual occurrence, and then, according to Hume's own showing, its non-occurrence would involve no contradiction, *or* else "analytical," and in that case it would have to be restricted to the content of a single idea, which clearly it is not. And so again, with reference to the argument that, since each impression may be treated as a unit and independently of its concrete setting, the relations of number may be regarded as at once certain and universal, it is clear that Hume is here making use of a distinction between the process of numbering and the things numbered to which on his theory he was not entitled, and which is indeed totally inconsistent therewith.

But, as we have seen, Hume certainly did not regard all mathematical propositions as analytical; and with respect to geometrical relations he completely fails to account for the universality and necessity which admittedly they seem to possess. Even the fundamental idea of equality he took to be based on, and to be a copy of, the crude impression of "equivalence," of which we have no other standard than that of immediate observation. And when he comes to explain what is meant in geometry by a line, surface, or solid, his failure to do so is no less apparent. Each of these is, he avers, a complex of "coloured points," so that one line is equal in length to another if it comprises the same number of "points." But these "points" are so minute and so confused with one another that there is no possibility of counting them. It is, indeed, difficult to treat such a statement seriously. If a line be a collection of "coloured points," it can only be

made up of coloured surfaces lying side by side, and separated, therefore, from one another. How could these constitute a line, or even a continuous surface? Not only so. Each "coloured point," or surface, would be divisible into parts, and these again into smaller parts, and so on indefinitely. Consequently, we should never come upon a line at all. In short, the line of the mathematician is obviously not made up of "points" of this description; it is continuous, and any attempt to reduce it to discrete elements is destructive of its very nature.

(c) Coming now to what we have called real relations, relations between matters of fact, we have in them evidently reference to an existent conceived as independent of the momentary state of perceiving. Whenever we assert that something is or that it was or that it will be, our thought seems to connect what is immediately present with an existent distinguishable therefrom and independent of it. The typical example of such propositions is to be found in those that are concerned with events that occur. It is, then, upon the relation of cause and effect that all propositions concerning matters of fact ultimately turn.

The analysis which Hume offers of the supposed connexion of cause and effect is well known. The result reached by him amounts virtually to a complete reversal of our ordinary conceptions of the relation of one event with another. According to his view, there is no reason in the nature of things why any one event might not be followed by any other event, no reason, for example, why the rays of the sun should not freeze water instead of converting it into vapour, or why the north wind should not set the world on fire. We have no other guarantee that this will not happen than a subjective belief, engendered by custom; but for aught we can tell to the

contrary, what we call "natural laws" are merely statements of accidental conjunctions, of the fortuitous arrangements of natural circumstances, in which there is no necessity of order or connexion to be found.

This view has been so often effectively criticised that I can here confine myself to three considerations. In the first place, I think it evident that Hume is really assuming that very necessity of connexion for the appearance of which he is professing to account. For we are clearly entitled to ask how it comes about that our "perceptions" do succeed one another in the regular, invariable way, which he asserts they do, in order to give rise to the *belief* or *feeling* of their necessary connexion. Hume recognises amply enough that, even if such necessity of connexion be a fiction, the task is imposed upon us of explaining how the fiction is produced. And it is produced, he contends, by the circumstance of our having repeatedly experienced that certain definite "perceptions" invariably occur when certain other definite "perceptions" have preceded them. But, if our "perceptions" do, as a matter of fact, succeed one another in this regular, invariable way, does not such invariability of succession really imply that very fact of necessary connexion which was called in question? In fine, in trying to explain how the *illusion* of necessary connexion is engendered, Hume is virtually admitting that necessary connexion is already there. The necessity, he declares, is a subjective feeling, a habit of expectation, and this subjective feeling or habit of expectation is formed by the invariability of the succession of our perceptions. Yet, this invariability which gives rise to the feeling or habit in question is, in truth, only conceivable as the expression of just that systematic order and connexion in the nature of things, of which Hume could discover no evidence. In the second place, it has to

be noted that a merely constantly observed sequence is never of itself sufficient to establish the fact of causal connexion. If it were, the relation between day and night would have to be regarded as a relation of cause and effect, for night constantly follows day, and the expectation we have on the experience of day that night will follow is as strong as any expectation can be. Because two events have been frequently, or even invariably, conjoined in experience, to conclude, from that circumstance alone, that they are related causally is a characteristic fallacy, a fallacy which logicians have signalised as that of *post hoc, ergo propter hoc*. No doubt a constant conjunction of two events does afford a presumption that the relation between them is a causal relation, but it is never in itself sufficient to establish that there is a causal relation. Only a thorough analysis of the conditions can give us a right to assert causal relatedness. Furthermore, it is not true that it is only as a result of repeated observation that we feel ourselves entitled to assert a case of causal connectedness. The procedure of the chemist or the physicist shows that his conviction of the uniformity of sequence is irrespective of the number of instances in which it has been observed. A single instance in which one event has been followed by another is sufficient to establish the fact of such uniformity, if it has been ascertained precisely what it is that precedes and what it is that follows *in that instance*. The scientist proceeds on the principle that what is a fact once is a fact always. And a uniformity which can be established in this way is precisely what is implied in the term "necessary." The fact of such uniformity is not contingent upon its having been experienced by anybody or everybody. It does not come into being with the experiment that brings about our awareness of it. It is experienced because it is real; it is not real because it is

experienced. Indeed, in actual scientific investigation it is often the case that what is directly observed is disregarded, and a hidden connexion is, by the experiment, established, in which the causal relation is found and acknowledged, although it differs from the observed features of the sequence. For example, when hydrogen and oxygen are combined in the proportion of two to one, and water is found to result from the combination, that connexion is forthwith regarded as necessary, and, accordingly, as a causal connexion. Nevertheless, the antecedent in this case only manifests itself as an antecedent when the experiment has been performed which produces the effect. It is obvious, therefore, that frequency of sequence has here nothing to do with engendering the belief in question. In the third place, I would call in question the assumption which lies at the root of Hume's whole theory of causation. So far from its being true that, as he asserts, there is nothing in any object considered in itself that can afford us a reason for drawing a conclusion beyond it, exactly the opposite thesis might be laid down and defended. A conscious mind, it might reasonably be agreed, is capable of recognising any object or event in itself; every object or event carries us inevitably beyond itself, and constrains us to recognise its connectedness with other objects or events. Without such recognition of connexion, apprehension of an object or of an event would be impossible. Causal connectedness can, then, never be resolved into mere conjunction or succession. No doubt, considered as an event, the effect may be looked upon as, in a sense, "distinct from" its cause. The point is, however, that it never is *merely* an event. To treat it merely as an event is to treat it as a mere particularity—that is to say, as a mere characterless unit. No effect is merely an event of that sort; it is

always an event possessed of qualities, an event of such and such a character. And this means that it is the same universal feature which is present in both cause and effect, and which unites them together as elements in a single whole. So far from its being true that "we are never able to discover any quality which binds the effect to the cause," it may be said that it is *precisely* the *same* quality which is present in the two particular events, and which constitutes their intelligibility.

That Hume was aware of the inherent impossibility of the task at which he had been labouring is apparent from his Appendix to the *Treatise*. "There are two principles," he says, "which I cannot render consistent; nor is it in my power to renounce either of them, namely, *that all our distinct perceptions are distinct existences*, and *that the mind never perceives any real connexion among distinct existences*. Did our perceptions either inhere in something simple and individual, or did the mind perceive some real connexion among them, there would be no difficulty in the case. For my part, I must plead the privilege of a sceptic, and confess that this difficulty is too hard for my understanding." With remarkable discernment and subtlety, Hume thus hits upon the essential weakness of the view of experience which he had himself propounded. He had done all that it is possible to do in the attempt to manufacture knowledge out of isolated mental states. He had tried to look upon what is known as consisting merely of disconnected particulars, connected only by external relations. And in the long run he has to admit that experience so conceived turns out to be as a rope of sand, that no ingenuity would avail to introduce coherence into elements defined from the outset as incoherents, or to bring into relation elements that had all along been regarded as relationless. I think

the value of Hume's philosophy largely consists in the thoroughness with which he endeavored to work out a thought which has always influenced philosophic speculation, a thought that had largely determined the theorising of Locke, but which had never previously been freed from extraneous considerations, and so had never been able to exhibit its real significance in regard to the conception of relations.

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F. H. BRADLEY ON IDEA AS IMAGE AND AS MEANING

IT often happens that success in a struggle bears with it a certain type of defeat. This is peculiarly true when the battle rages over principles and concepts, when ideas are the weapons and arguments the maneuvers. Something like this occurred in the revolt of British objective idealism against English associationism and empiricism. What I have in mind here is well illustrated in F. H. Bradley's forceful distinction between idea as image[1] or psychical state and idea as meaning. Such a distinction, especially when it becomes as fundamental as it is for Bradley, reveals an admission of failure on the part of the 'speculative idea' (*i. e.*, the concrete universal)[2] to account for the fact of irrelevance and brute particularity. Hence in Bradley's hands the conflict between the 'speculative' and the 'empirical'[3] ideas does not issue in a complete victory for the former. Rather, both leave the battlefield victorious and vanquished—but also transformed. The 'speculative idea'

[1] We might as well note at the outset the ambiguity of the term "image." Generally Bradley uses it loosely to cover all mental states. But sometimes he uses it in the narrower psychological sense as meaning centrally stimulated states as against peripherally stimulated states, or revived copies as against original sense perceptions. Unless the context indicates otherwise, we may interpret Bradley as using image in the wider sense.

[2] The "speculative idea" may be defined, in Hegel's terminology, as "a systematic totality which is only one idea, of which the several elements are each implicitly the idea . . ." (*Encyclopedia of the Philosophical Sciences*, § 243, Wallace's translation). This means, of course, to deny that ideas can ultimately be irrelevant, disunited, or merely conjoined facts of experience.

[3] By "empirical idea" I mean any object of thought as it is directly "had" by any human mind at any particular time.

has become a mutilated adjective, an abstract universal without a home in concrete fact. On the other hand, the 'empirical idea' still exists, but is reduced to a mental state, a mere psychological fact,[4] a fleeting existent which *is*, but is meaningless, and is neither true nor false.

What I wish to show is that this result is disastrous to absolute idealism, on the one hand, and that it is intrinsically untenable, on the other. The latter point, however, I do not mean to imply follows directly from the former. Both follow from a common doctrine which I assume to be true: viz., that both system or relevance and brute togetherness or irrelevance are irreducible, inseparable features of all of our ideas.

My intention is to consider the two-fold nature of idea for Bradley, first as it is expressed in *The Principles of Logic*, and then as it appears in Bradley's later writings. My conclusion will be a critical evaluation of Bradley's position in this matter.

It might be valuable as an introduction to indicate how Bradley's point of view was anticipated, to some degree, by Lotze. Lotze was interested in the movement by which thought attempts to transform mere conjunction of ideas in experience into rational connection. We find at the very outset of his *Logic*[5] an attempt to show how the logical concept is related to and develops out of psychical impressions, an attempt strikingly similar, in some ways, to that on the part of Bradley to develop meaning out of the image.

[4] The difference, I take it, between a "mental state" or "process" of modern introspective psychology and the "idea" of the English associationists is that the former is the product of abstract, scientific analysis and classification; the latter is the total unitary experience of any common-sense thing or object. However, this distinction is not accurate if taken too strictly. For the introspective psychologist often talks as though his mental processes were the total experiences we actually have on various occasions (especially in the psychological laboratory), and the associationist is willing to take the results of analysis of our ideas as themselves ideas. *Cf.* E. B. Titchener, *A Text-book of Psychology*, pp. 105-106.

[5] *Cf.* Bk. I, ch. i.

The first function of thought, Lotze tells us, is to objectify impressions, to convert them into ideas or logical building-stones. This it does by naming them, thus separating them, identifying them, and making them permanent and universal objects of reference. "As soon as we give the name of red or green to the different movements which waves of light produce through our eyes, we have separated something before unseparated, our sensitive act from the sensible matter to which it refers. This matter we now present to ourselves, no longer as a condition which we undergo, but as a something which has its being and its meaning in itself, and which continues to be what it is and to mean what it means whether we are conscious of it or not."[6] This does not mean, Lotze warns us, that such ideas or objectified impressions can exist outside all thought, but only that they are valid objects of reference for anyone at any time. "The logical objectification, then, which the creation of a name implies, does not give an external reality to the matter named; the common world, in which others are expected to recognise what we point to, is, speaking generally, only the world of thought; what we do here is to ascribe to it the first trace of an existence of its own and an outward order which is the same for all thinking beings and independent of them. . . ."[7]

In the second operation of thought (which is really the same as the first, but viewed from a different side), we have the three factors of positing (*Setzung*), distinction, and comparison of the content of ideas (*i. e.*, of impressions as objectified). Thought, here, does not bring external forms and superimpose them upon the matter of impressions. Rather, it simply recognizes the facts, interprets the relations holding between the impressions themselves in terms of aspects of their content. Thus we have what

[6] *Ibid.*, § 2 (Eng. tr., vol. i, p. 14).
[7] *Ibid.*, § 3 (Eng. tr., vol. i, pp. 15-16).

Lotze calls the 'first universal.' It is not conceptual, rather it is sensuous and is merely accepted by thought. It occurs as a common element in different impressions, its variations in the different impressions being quantitative only. The 'second universal' is the concept proper. It is an abstracted structure of essential marks. These marks are themselves, of course, universals, but are universals of the 'first' sort.

Such, in brief outline, is the way Lotze passes from impressions to concepts. Bradley does not follow Lotze in detail, but in many ways he seems to view his problem in the same light. First, he apparently agrees with Lotze that logic must start with wholly specific mental states. Again, it must interpret universal meanings as somehow developing out of these. Finally, it must interpret this development in terms of the 'matter' or 'content' of the mental states; that is, their specific existence is to be dropped out of sight in order that thought may get hold of permanent, universal predicates.

It seems to me unquestionable that in *The Principles of Logic* we have a distinct dualism, an emphasized split between idea considered as mental presentation of fact, on the one hand, and as meaning, on the other.[8] We are told repeatedly that the symbol or sign is not the symbolized and must not be confused with it, for the symbolized is distinctly something beyond, and the symbol, in referring to it, must point beyond itself and thereby lose its own individuality. And a mental fact is not as such a symbol. To be a psychical occurrence and to be a symbol are quite different things; apparently many of the former are not

[8] In Bradley's first work, *Ethical Studies*, one of his arguments against the pleasure-pain theory is that pleasure and pain are mental states; hence they are merely subjective and specific, and for that reason can not be set up as universal and objective standards. This would seem to imply that mental states are wholly subjective and specific; but perhaps Bradley is here accepting his opponent's views simply *ad hoc*. See *op. cit.*, 2nd ed., pp. 94-97.

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the latter. Hence we must sharply distinguish between the meaning of an idea and its existence. The meaning of an idea does not and can not exist. It is never a fact. The existence of an idea is never strictly either that idea's meaning or any meaning at all. And so we are presented with the following meaning of idea: as presented it never means, as meaning it is never presented. As meaning, idea is timeless and eternal; as presentation, it is always in flux. As meaning, it is always universal, in the sense of abstractly universal—exclusive of particularity and concrete detail; as presentation, it is always specific, particular, invested with its full quota of detail. As meaning, idea is adjectival; as presentation, it is substantive. As meaning, it is true or false; as presentation, it is brute fact which merely is.

Within idea as psychical presentation we are given a further distinction. It has content or quality, and it is an existence. These two, the 'what' and the 'that,' must be kept forever distinct. Now it is true that the content of a presentation or image is not as yet or as such meaning. But it can become meaning in the following way. If, instead of being taken in its concrete wholeness, abstraction of certain features of this content is made, if, further, this result of abstraction is fixed by the mind and, by the loss of concrete and detailed context, made universal, then we have meaning. Not all meaning need come this way—developing internally and by abstraction out of the content (as over against the existence) of a mental fact. For some meanings are, in source, quite external to that which acts as symbol or sign. And even when we have such an internally begotten meaning, we must remember that it is torn from all the detail and evanescence of its living context, and must, moreover, be stone dead—neither having

nor claiming any existence in the life of the stream of consciousness. Nevertheless Bradley insists that at least some meanings are simply mutilated abstractions from the content of images. Hence, though image and meaning are kept strictly distinct in Bradley's thought, yet we must accept the rather startling paradox that the universal meaning is nothing really other than the particular image —it is simply a mutilated part of the latter.

Such, in summary, is the effect of the impingement of the empirical idea upon the speculative idea, as revealed in *The Principles of Logic*. That is, the empirical idea loses its tacit meaningfulness and self-identity and becomes in earnest a mere particular—a mere, brute fact which never recurs. The speculative idea, on the other hand, tends to lose its concreteness; it is thrown back into a realm of abstract, adjectival, universal meaning. It may be true, it is always significant, but it can not exist. And though logical meaning is supposedly derived on occasion from psychical fact, still we are left wondering whether such a genesis is not more of a miracle than an indication of the fundamental unity of these two sorts of idea. Let us consider Bradley's own words.

"Every kind of fact must possess [the] two sides of existence and content. . . . But in anything that is a symbol we have also a third side, its signification, or that which it means."[9] "A sign is any fact that has a meaning, and meaning consists of a part of the content (original or acquired[10]), cut off, fixed by the mind, and considered apart

[9] *The Principles of Logic*. 2nd ed., vol. i, p. 3.

[10] Evidently Bradley uses the word "acquired" to cover the case where a sign stands "for that with which internally it is not connected, and with which it has been joined by arbitrary chance" (*ibid.*, vol. i, p. 4). It seems to me there is a real difficulty here for any interpretation that looks upon meaning as an abstraction from the given or presented, but Bradley passes it over as of no importance. Compare Stout's criticism below, pp. 613f.

It might not be amiss to note here that Bradley's view of sign and signified is not in harmony with Hegel's. For Hegel, the meaning is not simply a part of the content of the sign. "The sign is some immediate intuition, representing

from the existence of the sign."[11] Every idea "exists as a psychical fact, and with particular qualities and relations. It has its specialty as an event in my mind. It is a hard individual, so unique that it not only differs from all others, but even from itself at subsequent moments. And this character it must bear when confined to the two aspects of existence and content. But just so long as, and because, it keeps to this character, it is for logic no idea at all. It becomes one first when it begins to exist for the sake of its meaning. And its meaning, we may repeat, is a part of the content, used without regard to the rest, or the existence. I have the 'idea' of a horse, and that is a fact in my mind, existing in relation with the congeries of sensations and emotions and feelings which make my momentary state. It has particular traits of its own, which may be difficult to seize, but which, we are bound to suppose, are present. It is doubtless unique, the same with no other, nor yet with itself, but alone in the world of its fleeting moment. But, for logic, and in a matter of truth and falsehood, the case is quite changed. The 'idea' has here become an universal, since everything else is subordinate to the meaning. That connection of attributes we recognize as horse, is one part of the content of the unique horse-image, and this fragmentary part of the psychical event is all that in logic we know of or care for. Using this we treat the rest as husk and dross, which matters nothing to us, and makes no difference to the rest. The 'idea,' if that is the psychical state, is in logic a symbol. But it is better to say, the idea *is* the meaning, for existence and unessential content are wholly discarded. The

a totally different import from what naturally belongs to it; it is the pyramid into which a foreign soul has been conveyed, and where it is conserved." This is possible, Hegel says, because intelligence can treat its intuition of time and space "as its own property, deleting the connotation which properly and naturally belongs to it, and conferring on it another connotation as its soul and import" (*Encyclopedia*, §458, Wallace's translation).

[11] *Op cit.*, vol. i, p. 4.

idea, in the sense of mental image, is a sign of idea in the sense of meaning."[12] Psychical presentations "are so entangled with their environment, so one with their setting of sensuous particulars, that their character is destroyed if but one thread is broken. Fleeting and self-destructive as is their very endurance, wholly elusive their supposed individuality, misleading and deceptive their claim to reality, yet in some sense and somehow they *are*. They have existence; they are not thought but given. But an idea, if we use idea of meaning, is neither given nor presented but is taken. It can not as such exist. It can not ever be an event, with a place in the series of time or space. It can be a fact no more inside our heads than it can outside them. And, if you take this mere idea by itself, it is an adjective divorced, a parasite cut loose, a spirit without a body seeking rest in another, an abstraction from the concrete, a mere possibility which by itself *is* nothing."[13]

Bradley does not, in *The Principles of Logic,* treat us to a discussion as to how it is possible to mutilate a content of a presentation such that we end up with a fixed and universal meaning, nor as to what process we go through in doing this.[14] Nor are we enlightened as to how it is that a concrete image may be used as a sign for such an abstracted and non-concrete meaning. No doubt we would not ask for such discussions were we not left with the feeling that Bradley has been altogether too successful in

[12] *Ibid.*, vol. i, pp. 5-6.

[13] *Ibid.*, vol. i, pp. 7-8. Thus we are not so very surprised when we learn that "all meaning must be adjectival," or even when we discover that many of these adjectives are quite unattached and float about loosely. Bradley himself later recognizes the incompatibility of "floating adjectives" with the absolutistic point of view. However, even in his later thought he retains "content loosened from existence," which, it seems to me, still reveals the effects upon the speculative idea of the admission of the actual existence of the empirical idea (under the form of a meaningless mental state or psychical presentation).

[14] Bradley does, in a sense, discuss this problem in an article entitled, "On Pleasure, Pain, Desire and Volition," *Mind,* O. S., xii (1887), pp. 354-381.

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distinguishing idea as meaning from idea as fact. He has been so successful that we are inevitably left with the feeling that the natures of the two are quite foreign, and no matter how strongly it is asserted that in genesis one arises out of the other, still at heart they are not of one kin at all. We must not neglect the fact that Bradley often emphasizes that every meaning and every truth has its aspect of psychical presentation. But on the other hand, let us remember that meaning in its nature as meaning is never presented, and truth as truth can not exist. So it hardly seems that there is much gain here. We might well wonder how much unity the notion of aspect affords when that which is characterized as an aspect requires, by its own nature, the destruction of that of which it is an aspect, of that which is supposedly revealing itself in it.

There is one minor point we might notice before leaving *The Principles of Logic*. In judgment, we have seen, we use meanings, not psychical facts. It is due to the confusion of the former with the latter that the notion arises that in judgment there are two or more ideas together with the relations between them. It is true that distinctions can be drawn within a judgment, but it is false to conclude that the unity of the idea in judgment is thus destroyed, and that all that is left is a number of separate ideas and their relations. "The relations between the ideas are themselves ideal. They are not the psychical relations of mental facts. They do not exist between the symbols, but hold in the symbolized. They are part of the meaning and not of the existence. And the whole in which they subsist is ideal, and so one idea."[15] "We shall always go wrong unless we remember that the relations within the content of any meaning, however complex, are still not relations between mental existences. There is a wolf and

[15] *Op. cit.*, vol. i, p. 11.

a lamb. Does the wolf eat the lamb? The wolf eats the lamb. We have a relation here suggested or asserted between wolf and lamb, but that relation is (if I may use the word) not a *factual* connection between events in my head. What is meant is no psychical conjunction of images."[16] What, in short, we must remember is that in the realm of meaning differences do not destroy identity.[17]

Having considered Bradley's treatment of image and meaning in *The Principles of Logic,* let us now turn to relevant discussions in Bradley's writings subsequent to 1883. The distinction between psychical fact and universal reference or meaning, which was emphasized in *The Principles of Logic,* was later to a great extent resolved into the contrast between existence and content and the contrast between immediate feeling and thought. Thus we do not hear so much directly about image and meaning, though the basic dualism is still present in a slightly different form. We are still led to believe, nevertheless, that an idea (a meaning) is attained by the abstraction of a part of the content of a psychical fact from its whole concrete nature and also from its existence or presentedness.[18]

[16] *Ibid.,* vol. i, p. 12.

[17] The question arises: Do they in the realm of images? Despite Bradley's emphasis upon the particularity of presentations, it seems certain that for him psychical ideas (as well as logical) always have complexity in unity. At the lowest extreme they must have both "that" and "what," and the "what" must be complex, for out of it, by abstraction, meaning comes to be torn. Hence it would seem that Bradley can not admit, even in the realm of psychical images, that every difference is destructive of identity. Bradley's point, I think, is rather this. In the realm of meaning, there is no difference not expressive of an including identity, whereas in the realm of images, some differences, at least, are expressive of an ultimate non-identity or discreteness.

[18] An interesting and somewhat novel form of the contrast between the idea as logical content and psychical existence is found in a passage on the idea of an idea in "Is there any Special Activity of Attention?" *Mind,* O. S., xi (1886). "The idea of an idea is a psychical state, the character of which is used representatively and contains the feature of being an idea of a certain kind. We must distinguish two varieties. In the first of these the absent idea which I think of is the idea pure and simple, while in the second it will include my psychical state as I have this idea. For example, I possess a general idea of the solution of a problem, and that in the first case contains merely the general character of the answer required, or the principal feature of the necessary process. But if (as in the second case) I think of myself as having the

In several places Bradley warns us against confusing idea with image (in the psychological sense: image, that is, as contrasted with percept). Not only is the idea (the meaning) not an image, but often the symbol (the psychical fact that is used for or refers to the meaning) is not an image but an actual sensory experience. In a passage in an article entitled, "On Pleasure, Pain, Desire and Volition,"[19] Bradley brings this out, as well as the contrast between idea and psychical fact, and something of the genesis of idea. For instance he tells us: ". . . idea is no more equivalent to image than it is to sensation. With me the opposition here is not between external sensation and internal image, for both of these are mere psychical facts." Though image and sensation are different, they are still psychical facts and neither is an idea. For in neither is the content alienated from, indifferent to, independent of its existence. Thus we may have ideas without images, and images without ideas. Still every idea has its aspect of psychical fact. Hence some ideas work from or with content directly presented in sensation. Bradley points out three stages here: an idea which has not worked separate from its sensory basis though it does refer beyond it; a desired content presented in sensation which collides with the sensation, though not itself being presented as a separate image; and an idea which, in the form of a desired content directly in conflict with the sensation, gets presented in a distinct image. Thus the meaning is never an image, and does not always need an image (as distinguished from sensation) to represent it or serve as its starting place in the series of psychical fact.[20] In other

solution or as performing the process, I must represent also the psychical presence of the whole event, of course again only in its general aspect. Thus, if we realized the first idea we should have simply to fill out its logical content, but the reality of the second would give us its actual psychical existence" (*loc. cit.*, p. 313).

[19] *Mind*, O. S., xiii (1888), pp. 1 ff.

[20] *Cf. ibid.*, pp. 22-23.

places Breadley expresses the same warning against confusing idea with psychical image. "It is [commonly] believed that, in order to have something ideal which qualifies an object, we must have an image or images existing separate from that object. . . . But this identification of the ideal with images is surely a mistake."[21] When perception leads to action in low levels of mind, the idea is "the perceived object, so far as that is qualified inconsistently and qualified in a way that its meaning in part is made loose from itself. This meaning can therefore be applied as an adjective to a fresh subject."[22]

Idea, then, is not to be confused with psychical fact, nor particularly with image, for an idea never is a psychical fact, and though based on and using a psychical fact as a symbol, that fact need not be an image as distinguished from a percept. But on the other hand, Bradley tells us that the idea is always the same as the psychical fact in quality, and itself must occur as a psychical fact. This, of course, is in harmony with the doctrine that the idea is an abstracted portion of the content of the psychical fact which serves as symbol. "It should be a commonplace that ideas are psychical realities, and we cannot represent them without using a psychical fact. Further, what represents a pain must be a pain, and so again with pleasure."[23] "It is not wholly true that 'ideas[24] are not what they mean,' for if their meaning is *not* psychical fact, I should like to know how and where it exists."[25] "And right as it is to say with Lotze, that ideas are not what they mean, it is also true that ideas can mean nothing but that which they are. For unless a thing has, and to that extent is, a

[21] *Mind*, N. S., x (1901), p. 441.
[22] *Mind*, N. S., xi (1902), p. 462.
[23] *Mind*, O. S., xiii (1888), p. 9.
[24] Is "idea" here the psychical fact which represents the meaning?
[25] *Appearance and Reality*, p. 51.

quality, I cannot myself understand how it goes about to show it.[26] The idea of the extended has extension, the idea of the heavy has weight, the idea of the odorous has smell, and the idea of pleasure, beyond all controversy I should have thought, exists and is so far pleasant."[27] "But ideas and perceptions, I shall be told, are not what they signify. It is true, I reply, that their meaning and their existence are different. But if this difference is taken to preclude sameness, the statement would become false. . . . a feature of content, which makes the meaning of an idea, must, I presume, in order to do this, be present psychically."[28]

If the representing psychical fact and the represented meaning must be the same, the problem arises as to just how to interpret their difference. How, e. g., distinguish between the intensity or extensity of the psychical state and the intensity or extensity of that which is meant? There is, of course, recourse to the distinction between the world of existence and of truth, but Bradley does not play this up to any great extent in the present connection. We seem to be entering a region of some confusion here, but I think I note two types of interpretation. On the one hand, there is the distinction between symbol and meaning which claims that the symbol is richer in content than the meaning. The excess qualities of the symbol, irrelevant to the meaning, may differ in various ways. Thus they may differ quantitatively independently of the quantity of the content abstracted in the meaning. This interpretation Bradley uses in explaining how the strength or in-

[26] Bradley's footnote: "The existence of conventional and arbitrary signs, and of mere accredited representatives, forms no valid objection. For unless the meaning is *somehow* there and belonging to the sign, the sign would be no sign. The question is merely as to *how*, within the whole actual sign, the meaning and the existence are in each case connected—whether, that is, they are connected indirectly and through external association and credit, or directly and through an internal property."

[27] *Mind*, N. S., iv (1895), pp. 20-21.

[28] *Ibid.*, p. 225.

tensity of the psychical state may differ from that which is meant. "For in the being of an idea I find always psychical matter extraneous to its meaning, and this unemployed content, I must insist, can vary in quantity, while the specific meaning of the idea remains identical."[29] On the other hand, the psychical fact, the symbol, differs from what it means because it is abstract and incomplete, and leaves out some of the conditions of the latter. This seems to be Bradley's view when he is explaining the difference between the psychical extension of our mental states and the extension meant by them, i. e., the physical extension of the external world. "For when we perceive a sequence in nature we perceive in this but one feature of the whole. The result observed does not follow really and in fact except for complete conditions, and to experience these complete conditions is quite impossible. . . . We are right to deny that the physical extended in its full process has psychical existence."[30]

We may say, then, that the distinction between psychical fact and meaning still persists in Bradley's thought subsequent to 1883. The idea is not the image, nor any other fact, for it is abstracted content cut loose from existence. Yet, peculiarly, the meaning must be psychical fact, and must be in some respects the same as its symbol.

Let us turn from exposition to critical evaluation. My first point is that Bradley's sharp distinction between particular psychical occurrences and universal logical meanings is out of keeping with the teachings of absolute idealism. This point I believe should have become clear of itself through the previous exposition. The central principle of absolute idealism is that of the concrete universal, of systematic connection, of the immanence of the whole in

[29] *Ibid.*, p. 20.
[30] *Ibid.*, pp. 226-227.

all its parts. On last analysis, nothing is irrelevant to the speculative idea; every difference is included within it as an organic aspect of its own identity. Bradley's position undermines this in two ways. It admits, in the psychical fact, that irrelevant, merely particular ideas do exist. And, as a result of this admission, it accepts abstract universals. The first admission destroys the doctrine that every difference is an expression of a universal or whole of meaning. The second, overthrows the doctrine that every universal is concrete. Why, then, did Bradley ever allow the empirical idea to gain leverage in his system? First, no doubt, we have here a result of Bradley's bitter struggle against English empiricism—an imprint of the enemy's weapons. Again, Bradley probably felt that the empirical idea (in the rôle of a mental state) was basic to the science of psychology, a science which was just growing up and toward which Bradley had friendly feelings.[31] But finally, and I think fundamentally, it was Bradley's way of admitting the indubitable, experienced fact of irrelevant conjunction, of the externality of our ideas to one another.

This leads to my second problem. Apart from what absolute idealism may or may not demand, is Bradley's interpretation of image and meaning and their relation intrinsically tenable, or does it involve certain almost insuperable difficulties?

It was interesting to me to come across the following statement, published forty-four years after *The Principles of Logic*. "Separation of meaning from the psychological nature of ideas belongs to the most secure generalizations of modern philosophical science."[32] If this is so, then we must admit that all the zeal of F. C. S. Schiller has been

[31] See, *e.g.*, his article, "A Defense of Phenomenalism in Psychology," *Mind*, N. S., ix (1900), p. 26.

[32] Henry Lang in *The Open Court*, xli (1927), p. 385.

wasted! However, even in men worthy of serious consideration[33] we find a few hints of dissatisfaction with Bradley's treatment of the relation between image and meaning.[34]

Perhaps the most searching criticism of Bradley's doctrine of image and meaning is to be found in an article by G. F. Stout on "Mr. Bradley's Theory of Judgment."[35] It follows from Bradley's view of the relation of sign and meaning, Stout contends, that "whenever we think of or apprehend anything as having a certain qualification, the characters we have ascribed to it are wholly contained within our own psychical state at the moment."[36] This follows from Bradley's insistence that any meaning predicated is always a part of the content of the mental state which is the symbol. But Stout rejects this view. The content of the psychical state which functions as a symbol is often fragmentary in comparison with the idea (meaning), and sometimes wholly different from what we mean to predicate. "There are instances in which the specific nature of an object of thought as such does not seem to correspond even partially to any assignable content of our psychical state at the moment." This occurs, for example, in silent or spoken thought, where the symbols are conventional linguistic signs. It seems absurd to say that the meaning of a spoken word is a part of its content.[37]

[33] I have in mind here a passage by Bradley, *Essays on Truth and Reality*, p. 274.

[34] Henry Jones criticizes Lotze's doctrine that the datum of knowledge is a merely subjective state objectified by thought. Jones' objection is that if we start from mere subjective states, we have no way to get to knowledge, to meaning, to the objective (*cf. A Critical Account of the Philosophy of Lotze*, pp. 105-115). I think this criticism is sound, and I also believe that it applies to Bradley's image and meaning. E. P. Robins has pointed out ("Modern Theories of Judgment," in *The Philosophical Review*, vii (1898), pp. 583-603) that by starting from a psychical image, Bradley and Bosanquet follow the lead of such logicians as Sigwart and Lotze in admitting a wholly subjective datum from which thought starts. And hence they do not really escape subjectivism and representationalism.

[35] *Proceedings of the Aristotelian Society*, N. S., iii (1902-1903), p. 1.

[36] *Ibid.*, p. 6.

[37] *Cf. ibid.*, pp. 8-15.

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Again, Stout rejects Bradley's view that the idea predicated is always cut loose from the psychic state and used to qualify something else. If this were true, no one could ever become aware of the contrast between the content as a logical idea and as a part of the psychic state. "If, then, it is essential to judgment that the contents of immediate experience are cut loose, divorced, alienated, from their existence as contents of immediate experience,—if judgment merely consists in ascribing these 'wandering adjectives' to something else,—it does not seem possible that we should ever become able to predicate anything concerning immediate experience, either truly or falsely."[38]

R. F. A. Hoernlé, in an article on "Image, Idea and Meaning,"[39] largely follows Stout in his criticism of the theory of ideas as expressed by Bradley. It seems absurd to hold, Hoernlé urges, that logical meaning is a mere fragment of the psychical image. "On the contrary, it would be truer to say, that our meaning is *far fuller* than anything contained in the image."[40] It does not help matters to speak of 'fixing' a part of an image and of 'referring it away to another subject.' For whatever character we fix on is the particular character of that image, and is not, as such, referred to other subjects. For example, the redness of a given image is a particular red and is not the redness we predicate of some other subject. "The meaning lies beyond the image, and no operation on the image will get us to the meaning. And in any case it is unintelligible how this particular element of a particular image should lose its particularity by being 'referred away to another subject.' "[41]

The theory gains its plausibility, Hoernlé thinks, by

[38] *Ibid.*, p. 15. A psychical idea can not, for Bradley, be an idea of anything. But if so, Stout contends, it can not be an idea even for psychology.
[39] *Mind*, N. S., xvi (1907), p. 70.
[40] *Ibid.*, p. 73.
[41] *Ibid.*, p. 73.

holding that images[42] alone are signs. Otherwise it would be impossible to hold that there is a partial identity between the sign and its meaning. But how are we to explain meaningful experience in cases where there is defective imagery? Or, as Stout urges, how are we to account for meaning when the signs used are words? Hoernlé likewise agrees with Stout that all ideas (even 'psychological' ideas) are *of* something. "Indeed, I should be prepared to uphold the paradox that there is no idea, or combination of ideas, which is wholly meaningless."[43]

Quite similar in spirit to these criticisms by Stout and Hoernlé is a critical analysis of the problem of immediacy and meaning by Professor G. P. Adams.[44] The task Bradley set himself, says Professor Adams, was to develop all the distinctions of meaning, judgment, thought out of the characters to be found in our immediately felt psychical states. Bradley failed in this attempt, however, and he failed precisely because he started from a false initial premise. There is no wholly immediate experience, Professor Adams contends. All experience has its aspect of meaning; it comes in a context; it comes as relational and as having a reference beyond the content which is immediately present. Hence Professor Adams rejects the view that "the predicate is always a present content of immediate experience, even if that content be supposed to be used only as a logical meaning and not as a psychical image. The process by means of which the logical meaning is cut loose from the psychical image—and I should follow Bradley in recognizing such a process—is one in

[42] Images in the narrower psychological sense. We have noted that Bradley rejects Hoernlé's contention on this matter. But Bradley has not dealt with cases where there is (so far as we can see) no identity of content between the sign and its meaning. He avoids it by speaking of an "accrued" content of the image.

[43] *Ibid.*, p. 75.

[44] *Proceedings and Addresses of the American Philosophical Association*, 1927, vol. i, p. 109, reprinted from *The Philosophical Review*, xxxvii (1928).

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which the logical meaning, the predicate, acquires a content other than any content or character immediately experienced. The logical meaning is not solely a mutilated and impoverished fragment of felt immediacy; it is a richer content, saturated with characters which could never be compressed within the poor and momentary content of immediate experience."[45]

Personally, I find two chief difficulties in Bradley's treatment of image and meaning. The first is that I can not see how he gets two such radically disparate things into any sort of unity. The meaning is somehow developed out of the image. But just how can an objective, permanent, universal adjective be developed out of a wholly private, fleeting, particular fact? This I do not understand, and I am not helped by hearing that the mind 'fixes' and 'objectifies' the latter. Such vaguely suggested miracles may satisfy the superstitious, but they ought not to pass as adequate philosophical explanations. No mere specific, it would seem to me, can ever become universal. It may be possible that what is only implicitly universal may become explicitly so, or that what we mistake for a mere particular may, upon investigation and experiment, turn out to be universal. But this is a different matter. Furthermore, if an image or psychical fact is what Bradley (in *The Principles of Logic*) portrays it to be, it can not be a sign. Surely anything that functions as a sign must

[45] *Ibid.*, p. 125. For other criticisms of Bradley's theory of image and meaning, see W. C. Gore, "Image and Idea in Logic," in *Studies in Logical Theory*, by John Dewey and others, p. 184; and J. Laird, "The Nature of Ideas," in *Proceedings of the Aristotelian Society*, N. S., xxv (1924-1925), especially p. 254. Gore emphasizes the point that the Bradleyan analysis treats meanings alone (not images) as of logical importance. Gore objects to this. "It is only as the sense-contents of various images are discriminated and compared that anything like thinking can be conceived to go on. The particular sense-content of an image, instead of being a matter of logical indifference, is the condition, the possibility, of thinking" (*loc. cit.*, p. 201). Laird objects to Bradley's treatment of adjectives as images loosened from sensuous apprehension: ". . . it is quite impossible that images and predicates should be the same" (*loc. cit.*, p. 254).

itself be to some extent universal, for it is mediatory and referential. And so, in general, I feel that Bradley has split image and meaning so completely asunder that he can not really get them together again. Yet there is truth in his view that in psychical fact we have particularity and change, whereas in meaning we have universality, permanence, etc. Just what this element of truth is we shall inquire into presently.

My other chief difficulty with Bradley's views on image and meaning is that I can not agree that the meaning is a mere fragment of the content of the image (taken as sign). This criticism we have already seen to be urged by Stout, Hoernlé, and Adams, and so I need not pursue it further. It seems indubitable to me that what a sign means always lies beyond it, and hence is not wholly included in it. The signified must be external to the sign. On the other hand, it is true that the sign can not be *merely* external to the meaning. So far, I should agree, Bradley is correct. The sign is what it means.

As I see it, Bradley's difficulty lies right here. He sees that our ideas have an aspect of mere occurrence and particularity. But by sundering this from their self-transcendence, their universal meaning, he lays himself open to the charge of self-contradiction when he claims that after all meaning and image are the same.

The truth is, ideas are both particular occurrences and self-transcendent meanings. They are externally related to each other and they internally involve each other as elements in a systematic whole. But how unite these aspects? I should like to offer two suggestions. The first is that the distinction between idea as image and as meaning is always one of degree. It is never absolute. The second is that we can solve the present problem only if we distinguish between meaning and the meant.

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More definitely, my suggestion amounts to this. Both the psychical fact which serves as a sign and its meaning are aspects of an identity including them both, viz., the object meant. Hence image and meaning are one—they are features of a common identity. But they do not play the same rôle in this common identity. And it is this difference which makes it necessary to urge that the meaning is quite other than the idea taken as a mere fact or occurrence.

To make my point clear I shall have to digress long enough to make a dogmatic statement as to the nature of a real object or identity. A 'real' identity embraces all the characters truly predicable of the identity, no matter how trivial. But these characters are not all of equal importance to the identity. Some form a nucleus whose precise character is necessary to the identity. In contrast with this nucleus, there is always a mass of detail—irrelevant characters which are truly predicable of the identity, but which are not, *in their specific nature,* necessary to the identity (though that there be such a mass of detail is necessary). This mass of irrelevant detail furnishes the change and difference or 'non-identity' which is, as a *general* requirement, rationally necessary to the identity, and which is in its specific nature actually included in the identity, but which *is* not the identity, for it is not, in its *specific* character, essential to the identity. The identity would remain even though the specific character of this mass of detail were actually different. In brief, real identity both includes all its features and is focalized in or peculiarly restricted to a few of those features which form its core or focal structure.[46]

[46] Formal identity may be expressed: A = A; that is, in formal identity, differences are wholly cancelled or excluded from identity. Identity is ultimately reduced to identical elements which are simply no different. "Identity in difference" is ambiguous, but may mean simply: *abc* = *abc*; that is, every

The meant is always a total real identity, in all its concreteness. Our ideas are always partial or incomplete identities, which refer beyond themselves precisely because of this incompleteness. But simply because all our ideas are partial and thus self-transcendent, it does not follow that they are all equally so. An idea may be the very core of the meant, or, at the other extreme, it may be a very minor detail. In the former case, the idea is as permanent as the identity to which it 'refers.' In the latter case, it is a fleeting 'sign' which apparently has no stability. In the former case, it is obviously self-transcendent; it is the nature or essence of something more concrete and inclusive than itself. In the latter case, the tendency is to take it as a mere fact, as what it directly comes as (though, inconsistently, it is still 'of' something). In the former sense, we have the idea as a logical meaning. In the latter, we have it as an image or psychical fact. On this basis we can see why (as in Bradley's case) image and meaning should be so basically distinct as to be almost entirely external to each other. The psychical image is a mere detail of the identity meant, and hence it could have been otherwise and still have been a sign or image of the meant. But the logical meaning is the essential nature of the meant, and it could not have been different, the object meant remaining what it is. Let me urge, however, that this distinction is not absolute. Just as many if not most of the features of an identity are neither absolutely necessary to it nor wholly irrelevant factual details in it, so most ideas neither omit entirely all features essential to their object

identity includes all its differences as essential elements, and hence would be destroyed if any of them were inadvertently dropped out. The doctrine I suggest, of "focalized identity," may be symbolized: $bAc = dAe$. However, this symbolization is inadequate, for it does not show clearly that the identity does as a matter of fact include the changing details (bc and de) while yet (as focalized in its rational essence, viz., A) it is to be contrasted with this non-identity of detail.

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nor do they give us the complete focal structure of their object.

This point of view modifies the 'speculative idea' so as to admit the factor of externality emphasized by the 'empirical idea.' But the result is not a thorough split, as seems to be the case with Bradley, between existenceless, abstract, logical meanings and wholly factual, specific, psychical images. It rather, I think, points to the need of a fundamental rapprochement between the speculative and the empirical points of view.

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MR. RUSSELL'S THEORY OF PERCEPTION

SOME fifty years ago Lotze wrote: "it seems likely that after reaching a certain limit in the extent and depth of its enquiries, natural science will feel the need, in order to the possibility of further progress, of reverting to the task of defining exhaustively those centers of relation to which it had previously been able to attach its calculations while leaving their nature undetermined. In that case it will either originate a new Metaphysic of its own or it will adopt some existing system."[1] The third decade of the present century has witnessed the fulfilment of this expectation. Numerous attempts have recently been made to re-define those "centers of relation," and among these the writings of Professor A. N. Whitehead and Mr. Bertrand Russell are especially prominent. These attempts have inevitably involved a re-examination of the problems of epistemology, and the present paper will be confined to a discusison of certain features of Mr. Russell's views on these problems, with some comparison of his views with those of Professor Whitehead.

In his *Analysis of Mind* Mr. Russell wrote (pp. 97-8): "It was natural, though to my mind mistaken, to regard the 'real' table as the common cause of all the appearances which the table presents (as we say) to different observers. But why should we suppose that there is some one common cause of all these appearances? As

[1] *Metaphysics*, Bk. I, § vii.

we have seen, the notion of 'cause' is not so reliable as to allow us to infer the existence of something that, by its very nature, can never be observed." In his two recent works—*Outline of Philosophy* and *Analysis of Matter*—Mr. Russell appears to have adopted the view which he here so clearly repudiates and, in spite of all he has previously written about the notion of causation, he now makes it the very foundation of his philosophical doctrines, but without any satisfactory explanation of the reasons which have led him to this change, and without removing the doubts in which he had formerly held this notion. The reader is therefore left in considerable bewilderment, which is further increased by the unnecessarily paradoxical air of many of Mr. Russell's newer utterances, and the equivocal use of language in many of them.

Mr. Russell's general conclusions regarding our knowledge of what physicists suppose themselves to be studying are summed up in the following passages: (1) "The only legitimate attitude about the physical world seems to be one of complete agnosticism as regards all but its mathematical properties."[2] (2) "By examining our percepts we obtain knowledge which is not purely formal as to the matter of our own brains."[3] Mr. Russell also states that inferences regarding even the formal mathematical properties of "external matter" are "not demonstrative or certain." The question then presents itself whether, on this basis, the knowledge upon which Mr. Russell's theory of perception itself rests has the character the theory requires, or whether it is not in too much need of foundations to be capable of supporting such a superstructure.

We can begin by considering a passage which illustrates Mr. Russell's method and standpoint with special

[2] *Analysis of Matter*, p. 270.
[3] *Ibid.*, p. 382.

vividness: "What the physiologist sees when he looks at a brain is part of his own brain, not part of the brain he is examining. The feeling of paradox about this view comes, I should say, from wrong views of space. It is true that what we see is not located where our percept of our own brain would be located if we could see our own brain; but this is a question of perceptual space, not of the space of physics."[4]

The reader will probably agree that the paradox rests partly on wrong views of space, although he may have doubts about the ownership of those views. But he can hardly be blamed if he attributes it in part also to Mr. Russell's equivocal use of the word "see." First we are told that *all* the physiologist *can* see is part of his own brain, and then that *if* he *could* see his own brain the percept he would have of it would not be located where what he does see is located. It would be difficult to find a more needlessly confused passage than this in the whole of philosophical literature, or one which illustrates to greater perfection all those errors which Mr. Russell has so often called attention to in the writings of others. One would gather from it (if one can gather anything) that there are not only two spaces, but two brains and two meanings of the verb "to see," but what their connexion may be seems to be left in great obscurity. And yet Mr. Russell gives, as a reason for his view that "the whole of our perceptual world is, for physics, in our heads," the assertion that otherwise "there would be a spatio-temporal jump between stimulus and percept which would be quite unintelligible." Any other view, he says in the *Outline*, would involve a "preposterous kind of discontinuity." Thus the reader is encouraged in the hope that a difficulty previously felt is about to be removed, that something which

[4] *Ibid.*, p. 383.

has been unintelligible for previous theories is here at last explained. But what are we to say of the "intelligibility" of the seemingly miraculous jump from the space of physics to perceptual space which the "cause" is required to perform; and how are we to interpret the "discontinuity" between the brain which is *all* I *can* see and the brain I *could* see but don't?

Putting these difficulties aside for the moment and construing Mr. Russell's assertions as sympathetically as possible his view seems to be somewhat as follows: When a physiologist is said to be "seeing a brain" there is an entity *B* which is a bit of "external matter" and which he is (1) looking at, and (2) examining, but not (3) seeing. Of this he only knows the mathematical properties. But there is another entity *b* which he *does* see and which, although not part of *B*, is a part of another entity sufficiently like *B* to be called "another specimen of a *B*," and which is also part of the physiologist. The physiologist can also *examine* this *b* and thereby obtain non-formal knowledge of his own specimen of a *B*. Moreover he knows or supposes that the *b* he is seeing has been caused by the *B* of a minute fraction of a second ago in the sense that had that *B* not been where it was he would very probably not have seen *b*. But Mr. Russell apparently holds that all that is in any sense given consists of percepts, such as *b*, the rest, namely *B* and the causal relation, being at best doubtful assumptions, although not so doubtful, apparently, as they were when *The Analysis of Mind* was written.

The following passage also illustrates Mr. Russell's view and provides material for a more detailed analysis: "It is a fallacy to argue, as is sometimes done, that, if we cannot trust our senses, we cannot know that we have sense organs, or that there is any truth in physiology. If we find that several people, looking at Jones, see him just

as usual, while one person sees him looking queer; if the several see nothing queer in each other's eyes, while they all see something queer in the eyes of the one, in such circumstances, I say, it is natural and proper to correlate the two queernesses. The man who sees Jones differently from usual sees him through a medium which has an unusual effect; there is no more ground for scepticism than is to be derived from the effect of opera glasses. The sceptical argument is only valid as against naive realism."[5] Quite so; but how is such an argument possible on the basis of Mr. Russell's theory? Let us try to discover some of the essential presuppositions which this argument requires. This undertaking is confronted with a special difficulty created by the fact that Mr. Russell's theory apparently makes no provision for a "percipient." Percepts apparently just bob up in people's brains from time to time and get themselves labelled "queer" or "as usual" but there seems to be no provision for processes of perceiving or comparing. But as Mr. Russell himself speaks of someone "looking at Jones" and "seeing him just as usual" it is presumably permissible to refer to that person as a "percipient" without making any metaphysical assertions about his "nature."

The first point to mention is, of course, that no one does in fact see Jones, according to Mr. Russell. Let us say then that one man A sees a percept J which (1) is a part of A's brain, and (2) is causally dependent upon an entity called Jones and in virtue (partly) of this relation is called a percept "of" Jones. (3) This percept J is compared with at least one other percept seen by A on a previous occasion when it was also a percept "of" Jones, the result of this comparison being that J is pronounced to be "just as usual." Next let B be another observer who participated in the above incident. He sees a percept J' which

[5] *Analysis of Matter*, p. 262.

(1) is a part of *B's* brain; (2) is causally dependent upon the *same* entity called Jones as *A's J* was dependent upon and is also called a percept "of" Jones. (3) *J'* is also compared with previously seen parts of *B's* brain which were percepts "of" Jones, and pronounced to be "just as usual." Finally, a third observer *C* sees a percept *J''* having properties analogous to *J* and *J'* of the brains of *A* and *B*, but with the important difference that when *J''* is compared with previous percepts "of" Jones it is found to "look queer."

Now *A* also sees two percepts *E*b *and E*c such that (1) each is part of *A's* brain; (2) *E*b is a percept "of" a part of *B*, namely *B's* eye, and *E*c is a percept of a "corresponding" part of *C;* (3) a comparison of *E*b *with E*c and of each with previously seen percepts of corresponding parts of *B* and *C* respectively leads to *E*b being described "as usual" and *E*c being described as "queer." *B* sees analogous percepts *E*a′ and *E*c′, such that *E*a′ resembles *E*b in being "as usual" and *E*c′ resembles *E*c in being "queer." Also *C* sees analogous percepts *E*a″ and *E*b″ but *both* "look queer," and consequently *E*a″ differs from *E*a′ and *E*b″ differs from *E*b.

Now in any causal investigation we always have a *comparison* of two situations, and if they are found to differ with respect to the determinate characters manifested under a given determinable then the causal postulate leads us to assert that there is *another* pair of differences correlated with the pair observed, even although the second pair is not observed. We are led to expect a *symmetry* of pairs of differences. Let us suppose that *A* is conducting the inquiry on the present occasion. Let us assume that he knows that *J* differs from *J''*, then he will argue *either* that the Jones of which *J* is a percept is not the Jones of which *J''* is a percept, or that there is a difference between some

other causal elements in the situation. *If* now he *also* knows that *E*a″ differs from *E*a′ and that *E*c differs from *E*b, but not from *E*c′, and *if* he knows that *J* and *J*″ both depend upon the entities of which *E*a″, *E*c, and *E*b are percepts, then *if* he *also* knows that the Jones of which *J* is a percept is *not* different from the Jones of which *J*″ is a percept, he may conclude that the difference between *J* and *J*″ is correlated with a difference between the eye of *A* and the eye of *C*.

Summarizing the requirements for this argument we have at least the following:

(1) The percepts of any given percipient, *A* say, fall into certain classes as follows. If α is a class of percepts of *A* then there is (in Whiteheadian language) some historical route *r* every temporal stretch of which has at least some properties in common with every other stretch, and every member of the class α of *A's* percepts is a term in a many-one relation to this historical route (e.g., the route pervaded by Jones).

(2) It must be possible for the members of a class of percepts of B to stand in a similar relation to the same historical route *r* as that in which the members of α stand to it. And it must be possible for *A* to know when this is the case.

(3) It must be possible for a percipient to compare one percept correlated with a given route with a previously perceived percept correlated with the same route, although of course with a different part of it.

(4) It must be possible for a percipient to compare a percept correlated with one route with another percept correlated with a different route.

(5) It must be possible for a given percipient such as *A* to know that a certain percept *p*′, of another percipient such as *B*, which is correlated with a certain route *r* is

different (or not different) from his own percept p which is correlated with approximately the same stretch of the same route r.

(6) The following inference must be possible for such a percipient as A: If P and P' are two percepts of different percipients but both correlated with approximately the same part of the same historical route r, and such that P is qualitatively different from P'; and if p and p' are two other percepts, of the same two percipients, such that p is correlated with an historical route r' and p' with another route r'', and p is qualitatively different from p'; and if the character of P is dependent upon r' and that of P' on r'', then the difference between P and P' is causally correlated with the difference between r, and r''.

Now according to Mr. Russell *A's* knowledge is confined to (1) the mathematical properties of the external world, with complete agnosticism regarding any other properties it may have; and (2) "knowledge" which is not purely formal about the matter of his own brain obtained by an inspection of his own percepts. But requisite (5) above is absolutely essential for Mr. Russell's argument. It is essential that *A* should know that his *J* resembles *B's J'* in "looking just as usual," and that his *Ec* resembles *B's Ec'* in "looking queer." But now *B's Ec'* is a percept and hence, according to Mr. Russell a part of *B's* brain, and if *A* knows that it looks queer, and if looking queer is a non-formal property, it follows that *A* has knowledge which is not purely formal of the external world, since *B's* brain is a part of the external world of *A*. Thus, although *A* cannot see *B's* brain he knows something about it which he cannot know by looking at it or "examining" it, and this knowledge is essential for Mr. Russell's theory of perception, although it does not seem to be provided for by Mr. Russell's conclusions.

Another important point which does not seem to be provided for, having regard to the doubts attaching (according to Mr. Russell) to our knowledge of even the mathematical properties of "external matter," is the following: If percepts are the kind of entities that can "be in" heads, if our percepts *are* in our heads, and if we *know* this fact about them, then clearly we have some *other* source of knowledge over and above that provided by the bare "examination" of our percepts, as Mr. Russell describes them. For we do not observe a spatial relation between our heads and our percepts on this theory. We see our percepts but we do not see our heads. Our alleged knowledge *that* we have heads, and *that* our percepts are *in* them, we must owe to some other source. A percipient who never perceived anything but percepts in his head would clearly have no ground for believing (a) that there *were* brains, and (b) that the entities he perceived were parts of one of these brains. Thus Mr. Russell's theory not only demands that A should know that C's percept J'' is different from his own percept J, but also that both of these percepts relate to the same entity Jones which is not a percept. Mr. Russell makes no real provision for this.

We must consider what it can possibly *mean* to say that "The whole of our perceptual world is, for physics, in our heads." How do we come by the notion of "being in"? Let us consider the proposition: "A match-box is *in* that room." This is a proposition which would ordinarily be taken to assert a fact, not about brains, but about what is called the "physical world," a fact, moreover, belonging to a large class of facts which have the peculiarity of being *public*. The number of percipients who could know this fact is limited only by the number of people who can be packed into the room, or who can crane up their heads far enough to see into the windows. The essential point about

the proposition so far as "in" is concerned is roughly as follows: If the whole three-dimensional spread of what is called the room is exhaustively analysed into a number of equal volumes then a certain number of these volumes suitably chosen will also be an exhaustive analysis of the three-dimensional spread of what is called the match-box. No possible analysis of the latter will yield a volume which is not also a volume obtained by an analysis of the room. Thus "being in" is, in this case, a relation between two three-dimensional entities (time being the same for both), which is theoretically capable of being apprehended by any number of percipients. But Mr. Russell appears to make no provision for any such apprehension. All that his percipients can perceive are parts of their own brains.

But now, can a percept stand in the above relation of "being in" to anything? Can a percept be a value of x in the function "$\hat{x}$ is in a head"? If not, the assertion that percepts are in heads is not a proposition at all but a meaningless string of noises or print-marks. When I look at the match-box I seem to perceive a relation between the room, the match-box and my body—these entities being marked out from one another by a display of coloured spreads. Three such spreads distinguish the match-box. The top is yellow, one side is blue, and another is black. Also I find that by stretching out my arm I can experience certain tactual percepts in virtue of which I call the box "tangible" or "solid." But it would be perfectly possible, by a suitable contrivance, to perceive the display of coloured spreads without any tactile experience being correlated with the *same place*. Thus coloured spreads may *appear in* places without being correlated with other percepts, but they always *appear in* some place, and *that* place always *is in* another place of greater spatial extent. This

suggests that "appearing in" a place is a different relation from "being in" a place. It is only the three-dimensional volume of the match-box—marked out for perception by the coloured spreads—which stands in the relation "being in" to the volume of the room. Moreover the relation "appearing in" involves a reference to a percipient, whereas the relation "being in" does not. But if this is the case a percept can never be a value of x in "x is in a head." Neither can a percept stand in the relation "appearing in" to a head, with the possible exception of head-aches, if the head is the head of the percipient. All percepts of that percipient stand, for him, in the relation "appearing in" places outside his skull. If we are to make any sense at all of the bulk of what passes for knowledge, it is necessary that we should in some way, and in some sense, be able to perceive places outside our skulls, so that A in our example can know that his percept J is a percept "of" the same Jones of which C's percept J'' is a percept. And any theory of knowledge which does not enable us to make sense of what passes for knowledge may reasonably be asked to explain the success of this common knowledge, but *without appealing to it itself,* and this is not usually done.

Now according to Mr. Russell the space we perceive is perceptual space, and this *is in* our skulls and our skulls *are in* the space of physics. But what does this mean when we translate it into modern terms based on the recognition that space is derived by a process of abstraction from a four-dimensional continuum? What grounds are there on Mr. Russell's theory, for speaking about any space at all other than perceptual space? Mr. Russell himself furnishes a hint towards an answer to these questions in a passage in his *Outline of Philosophy,* although it is only a hint and does not seem to be provided for in the theory

itself. After explaining that modern physics "reduces matter to a set of events which proceed outward from a centre" he adds: "if, therefore, we see red and blue side by side, we are justified in inferring that *in the direction* where we see red something different is happening from what is happening in the direction where we see blue." The words I have italicized seem to suggest that there is, after all, a way out from perceptual space to the space of physics. But according to Mr. Russell this direction will be a direction in perceptual space. Only if the direction which I immediately apprehend is itself a direction in the "space of physics" will it be possible to make the inference Mr. Russell makes. Thus whatever may be the case about the spatial relations of percepts (in the sense of "being in"), assuming they have any, it must be possible to "see" or in some way apprehend at least the *directions* of "the space of physics," even in order to obtain the basis for Mr. Russell's theory.

Thus in the ordinary accounts of perception as represented by Mr. Russell something essential is not officially allowed for but is introduced by vague hints and back-door methods—exposing such theories to the charge of inconsistency, or of arriving at a feeble scepticism which no one takes seriously. Among modern writers Professor Whitehead seems to stand almost alone in his method of dealing with this missing ingredient and in his persistent rejection of all theories which fail to recognize it. A fundamental tenet of Prof. Whitehead's doctrine is the recognition of two totally different elements in nature: four-dimensional spatio-temporal entities which he calls events, on the one hand, and non-spatial, non-temporal entities called objects, on the other. He has also perceived that the relation between certain objects (which include Mr. Rus-

sell's percepts) and events is a much more complex one than we have been led to suppose under the influence of the Aristotelian notion of substance and attribute standing in a two-termed relation to one another. Moreover the relation between objects and events is different from the relations between events—thus providing a basis for the difference between "appearing in" and "being in." In all his books Prof. Whitehead has urged the two-fold aspect of perception. In the *Principles of Natural Knowledge* there was apprehension of events and recognition of objects. In the *Concept of Nature* there was the difference between the discerned and the discernible. In the *Principle of Relativity* we have "cognisance by adjective" and "cognisance by relatedness." "Your knowledge is not merely of something which is red. The patch is *there* and it endures while you are observing it. Thus you are cognisant of it as having spatio-temporal position, and by this we mean a certain type of relatedness to the rest of nature which is thereby involved in our particular experience."[6] In Prof. Whitehead's last two works—*Symbolism* and *Process and Reality*—a corresponding distinction is made and worked out between the "two perceptive modes" of "presentational immediacy" and "causal efficacy" with an interplay of "symbolic reference" between them. Mr. Russell's theory provides only for perception under the first mode, leaving physics and physiology (upon which it professes to base itself) with no place in the scheme at all, since they deal with hypothetical entities in a hypothetical causal relation. As Prof. Whitehead puts it: "the physiological explanation remains, from the point of view of Hume's philosophy, a tissue of irrelevancies. It presupposes a side of the universe about which, on Hume's

[6] *Principle of Relativity*, p. 62.

theory, we must remain in blank ignorance."[7] Those who follow Hume and Mr. Russell are committed to a perpetual wavering in their attitude towards natural science. There is a passage in which Prof. Whitehead describes the part played by these two aspects of perception in scientific observation so aptly that it will perhaps be permissible to quote it in full: "The respective rôles of the two perceptive modes in experience are aptly exemplified by the fact that all scientific observations, such as measurements, determinations of relative spatial position, determinations of sense-data such as colours, sounds, tastes, smells, temperature feelings, touch feelings, etc., are made in the perceptive mode of presentational immediacy; and that great care is exerted to keep this mode pure, that is to say, devoid of symbolic reference to causal efficacy. In this way accuracy is secured, in the sense that the direct observation is purged of all interpretation. On the other hand, all scientific theory is stated in terms referring exclusively to the scheme of relatedness, which so far as it is observed, involves the percepta in the pure mode of causal efficacy. It thus stands out at once, that what we want to know about, from the point of view either of curiosity or of technology, chiefly resides in those aspects of the world disclosed in causal efficacy: but that what we can distinctly register is chiefly to be found among the percepta in the mode of presentational immediacy."[8]

Finally, objection may be made to Mr. Russell's assertion that an examination of our percepts yields us *knowledge* which is "not purely formal" concerning "the matter of our own brains." This assertion as it stands cannot possibly mean what it says. Otherwise it would indeed be strange that physiologists and anatomists should, in order

[7] *Process and Reality*, p. 246.

[8] *Process and Reality*, p. 238. *Cf.* the whole of Part II, Ch. viii.

to obtain such knowledge, have devoted long laborious lives to the study of the brains of other people, after subjecting them to tedious and complicated technical processes; and that they should be so mistaken about the character of the knowledge thus obtained, when it could have been reached equally well by the examination of their own percepts of bath-chairs or fire-irons. Surely the only *knowledge* that I can be said to obtain by the bare examination of my percepts as such is that they are sometimes, say, round and red, or smooth and cold, and so forth. I cannot, without further data, pass from this to the assertion that my brain is round and red or smooth and cold. I must first learn, not by "examining my percepts" in Mr. Russell's sense, but by exercising *both* perceptive modes, that there are spatio-temporal entities called brains. I must next be able to infer that I have a brain and that when I am sensing a red patch I am sensing a red part of my brain. But this is not *knowing* my own brain. From the point of view of *knowledge,* as contrasted with bare *sensing,* my own brain is not in any privileged position. Since I get knowledge of *other* brains first, and since I reach this knowledge by using my percepts (not my percepts as such but only certain ones known to be "of" other brains) this process of "examining" my percepts cannot be said to give any information of special merit in relation to my own brain. Mr. Russell's paradoxes seem to rest very largely on his confusing of sensing and knowing. My contention is that sensing percepts and knowing brains are two totally different processes. Mr. Russell appears to be urging that we can have qualitative knowledge only of our own brains, when it would be more correct to say that we can have such knowledge only of our percepts, e.g., that they are sometimes red. We are not entitled, without further ado,

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to pass from this to the assertion that this is *knowledge* about the matter of our brains. There is, moreover, another difficulty. Mr. Russell says that physics has "reduced" matter to a "set of events which proceed outward from a center," but he gives no explanation of the relation between the brain in the guise of "matter" as a set of events proceeding from a center, and the brain in the guise of, say, the smell of sulphuretted hydrogen. The latter does not seem to have the character of a set of events proceeding from a centre, and yet, according to Mr. Russell, when I smell it I am having non-formal knowledge of my brain which is matter and hence a set of events proceeding from a center. Between these two aspects of the brain there is, even in Mr. Russell's theory, a "preposterous kind of discontinuity" which is "quite unintelligible." This discontinuity is not, therefore, overcome after all by identifying our percepts with parts of our brains and putting them in our heads. Professor Whitehead's theory of objects and events does not attempt to overcome this antithesis. We are left with the relation of "ingression" between them. Our world persists in falling asunder in this way, leaving the relation between sense-objects (or percepts) and other entities as a thorn in the flesh of monistic philosophers.

J. H. Woodger

London.

FLORIAN CAJORI

February 28, 1859-August 14, 1930

IN the death of Professor Florian Cajori the world has lost one of the best-known of its recent historians of science, not merely in the domain of mathematics but in the contiguous domains of physics, geodesy, and to a certain extent astronomy. Beginning his contributions to the history of mathematics while he was professor of physics at Colorado College, his first important work, *The Teaching and History of Mathematics in the United States*, appeared in 1890, when he was thirty-one years old. This was followed four years later by *A History of Mathematics* (1894), and after two years more by *A History of Elementary Mathematics* (1896). Of these two, the former was revised in 1919, and the latter in 1917. During his nine years in the department of physics (1889-1898) he became so deeply interested in the development of the subject that he wrote a *History of Physics* which was published (1899) the year after he assumed the duties of professor of mathematics (1898-1918) in the same institution. In this period he added to his duties those of dean of the college of engineering (1903-1918), also finding the time and opportunity for beginning his real work as a student of source material. To this he was encouraged by the honor of a call to contribute to volume four of Cantor's well-known treatise, the *Vorlesungen über Geschichte der Mathematik* (Vol. IV,

1908). This led him to the study of the development of arithmetic during a definite period and to a consideration of sources not so essential in his earlier works. The spirit thus awakened showed itself in the *History of the Logarithmic Slide Rule* (1909), his most valuable contribution up to that time. This in turn led him to the study of the lives of individuals connected with this instrument, among whom was the subject of his first biographical work, *William Oughtred* (1916), one of the leaders in the popularizing of mathematics in the first half of the seventeenth century.

Not only had Professor Cajori now found his place, but the place now found him. In 1918 he was called to the University of California as professor of the history of mathematics, doubtless the first man in the world to hold this title and be given the privilege of devoting all of his time to the subject. It was a great opportunity and the appointment was fully justified in the twelve active years that remained. In this period were published the *History of the Concepts of Limits and Fluxions in Great Britain from Newton to Woodhouse* (1919), the *Early Mathematical Sciences in North and South America* (1928), *The Chequered Career of Ferdinand Rudolph Hassler, First Superintendent of the United States Coast Survey* (1929), and the *Magnum Opus* by which he will be most widely known, the *History of Mathematical Notations* (2 volumes, 1928, 1929).

Not only was he a writer of books, but he was an indefatigable contributor to scientific journals in this country and abroad. The number of these journals and of his monographs is impressive and his numerous articles will long be looked upon as material from which historians will draw.

Born in Switzerland, coming to this country at the age of sixteen, educated at the University of Wisconsin (B. S.

1883) and Johns Hopkins (1884-1885), he began his teaching as assistant professor of mathematics at Tulane University in 1885, continuing at Colorado College (1889-1918), and closing his labors at the University of California (1918-1930). As to his work in these institutions, as to the academic honors bestowed upon him (Ph. D., Tulane, 1894; LL. D., University of Colorado, 1912, and Colorado College, 1913; Sc. D., Wisconsin, 1913), and as to his membership and official positions in various learned societies it is not possible, in the space here allowed, to speak in detail. Suffice it to say that his work was faithfully performed and the honors were worthily bestowed. Standing as the leading historian of mathematics in this country, his loss will be deeply felt by all who knew him, whether personally or through his contributions to the subjects of his major interest.

DAVID EUGENE SMITH.

SOME RECENT BOOKS

Proceedings of the First Indian Philosophical Congress Held in Calcutta University, December 19-22, 1925. Edited by Satischandra Chatterjee. Calcutta, The Calcutta Philosophical Society, 1927. Pp. xxiii + 469. 10s.

Even though this volume of Proceedings omits such papers read at the Congress as have elsewhere been published, the material is so rich that one cannot but be impressed with the quality and the range of philosophic thought in present day India. In addition to Rabindranath Tagore's presidential address, the Proceedings contain eighteen papers in the field of Indian philosophy, ten papers on logic and metaphysics, eight papers relating to the philosophy of religion, four papers read in the Section of History of Philosophy, and three in the Section of Ethics and Social Philosophy. The volume should be made accessible to all serious students of philosophy.

A Debate on the Theory of Relativity. By Robert D. Carmichael, Harold T. David, William D. MacMillan and Mason E. Hufford, with an introduction by William Lowe Bryan. Chicago, The Open Court Publishing Company, 1927. Pp. viii + 154. $2.00.

This debate, in which the first two authors mentioned (mathematicians from the University of Illinois and from Indiana University) argued in favor of the theory and the last two (an astronomer from the University of Chicago and a physicist from Indiana University) opposed the theory, gives to the intelligent reader an unusually lucid and interesting account and appraisal of the work of Einstein. The debate was originally arranged by the Indiana chapter of Sigma Xi, where it was staged shortly before the manuscripts were made available in print.

The Public and its Problems. By John Dewey. New York, Henry Holt & Co., 1927. Pp. vi + 224.

In this book Professor Dewey scrutinizes our democratic state, the historical factors upon which its emergence depends, the sources of its present failure to function, and the indispensable conditions for its realization. Of additional interest are significant redefinitions of such concepts as "individual" and "social," "private" and "public," and the "state"—all of this in the light of the writer's "consequence" theory. Though constructive as well as critical, this study does not regard as significant or legitimate the search for a concrete program purporting to bring political salvation.

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